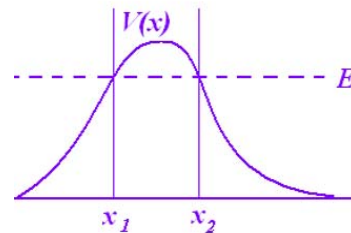


Problem Set #6

19. A particle with energy E and mass m is incident from the left on the potential barrier $V(x)$.



- (a) What conditions as $x \rightarrow \pm\infty$ should be imposed on the energy eigenfunction $\psi_E(x)$ appropriate to this problem?
- (b) Use the WKB approximation to obtain an expression for the probability of finding the particle to the right of the barrier at a much later time t for the case that E is less than the maximum value of V . Be careful to use only the information that is available from the known asymptotic form of the solution, avoiding the addition of elements of the solution which are not known without a more complete solution.
20. Consider the simplified Schrödinger equation:

$$-\frac{d^2}{dx^2} + V(x)\psi(x) = E\psi(x)$$

where $V(x)$ is given by:

$$V(x) = 20e^{-(x-50)^2/48}$$

which describes a barrier penetration problem. Assume that a plane wave with amplitude e^{-ipx} is incident on the barrier from the right, where $p = \sqrt{E}$.

- (a) Find the solution to this equation in the region $0 \leq x \leq 100$ for $E=16$ numerically. Plot your results for the real and imaginary parts of $\psi(x)$.
- (b) Find numerical values for the magnitude and phase of the reflected and transmitted waves. Use a step size $Dx = 0.005$ and the leap-frog method implemented in the [Schrodinger.ipynb](#) example posted on the web.

For your solution to this problem you need only submit the plot obtained from part a) and the four numbers obtained in part b). [It may be helpful to begin with a working Jupyter notebook which can be downloaded from the course web site, [Python](#), which solves the case $V(x) = 0$ and then to modify this sample program to solve the problem here.]

21. Find the three matrices $(J_i)_{mm'} = \langle j, m | J_i | j, m' \rangle$, $1 \leq i \leq 3$, explicitly for the case $j = 1$. Verify the commutation relation: $[J_1, J_2] = i\hbar J_3$.