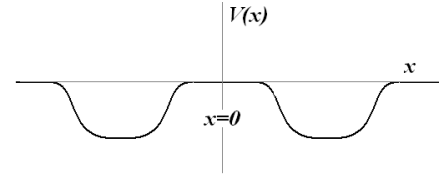


## Problem Set #8

26. Consider a one-dimensional potential  $V(x)$  of the form shown, symmetric about  $x = 0$ :  $V(x) = V(-x)$ .



- (a) Prove that the eigenstates of the Hamiltonian

$$H = \frac{\partial^2}{\partial x^2} \frac{\hbar^2}{2m} + V(x)$$

can be chosen to obey  $\psi(x) = \pm\psi(-x)$ .

- (b) If the two potential wells are far separated then the normalizable eigenstates are approximately the superpositions:

$$\psi_{n,\pm}(x) = \phi_n^L(x) \pm \phi_n^R(x)$$

where  $\phi_n^L(x) = \phi_n^R(-x)$  is a normalizable eigenfunction for the potential

$$V'(x) = \begin{cases} V(x) & x < 0 \\ 0 & 0 < x. \end{cases}$$

Use the WKB approximation to find the difference in energy of  $\psi_{n,+}$  and  $\psi_{n,-}$  for the case of a large but finite separation between the wells.

27. A spin-1/2 particle with magnetic moment  $\vec{\mu} = \gamma\vec{s}$  is placed in a magnetic field  $\vec{B}_0 = B_0\hat{z}$  in the  $z$ -direction. Use eigenstates  $|\pm \frac{1}{2}\rangle$  of  $s_z$  with eigenvalues  $\pm\hbar/2$ .

- (a) For an initial state  $|\psi(0)\rangle = a_+|+\frac{1}{2}\rangle + a_-|-\frac{1}{2}\rangle$ , find the resulting state  $|\psi(t)\rangle$  at a later time  $t$ .
- (b) Next a time-dependent magnetic field

$$\vec{B}_1(t) = B_1\hat{x}\cos(\omega t) - B_1\hat{y}\sin(\omega t) \quad (1)$$

is added to the original field  $\vec{B}_0$ . Solve the resulting time-dependent Schrödinger equation for these combined magnetic fields by working in a “rotating” frame: deriving and solving the Schrödinger equation obeyed by the transformed state:

$$|\psi'(t)\rangle = e^{-i\omega t\hat{z}\cdot\vec{s}/\hbar}|\psi(t)\rangle. \quad (2)$$

[Note: while it offers insight to view this as a transformation to a rotating frame, Eq. (2) can be viewed as a simple mathematical transformation which makes the problem easy to solve without any need for physical intuition.]

- (c) Describe the resulting motion of the particle’s spin as  $\omega$  is varied through the Larmor precession frequency  $\omega_L = \gamma B_0$ .