Physics G6037 Professor Christ

due 11/16/2018

Problem Set #9

28. Consider the Hydrogen atom as a two-particle problem which depends on the two, 3-dimensional coordinates $\vec{r}_e$ and $\vec{r}_p$, the locations of the electron and proton.

    (a) Write down the quantum mechanical Hamiltonian in terms of the positions $\vec{r}_e$ and $\vec{r}_p$ and their conjugate momentum operators $\vec{p}_e$ and $\vec{p}_p$.

    (b) Working with the resulting time-independent Schrödinger equation in position space, change from the coordiates $\vec{r}_e$ and $\vec{r}_p$ to the relative and center of mass coordinates
$$\vec{r} = \vec{r}_e - \vec{r}_p, \quad \vec{R} = \frac{1}{m_e + m_p}\Big\{m_e\vec{r}_e + m_p\vec{r}_p\Big\}$$
and rewrite the stationary state Schrödinger equation in terms of these variables. Here m_e and m_p are the masses of the electron and proton.

    (c) Show that the allowed energies of the hydrogen atom become:
$$E_n = -\frac{m_e e^4}{2\hbar^2}\frac{1}{n^2}\frac{1}{1+\frac{m_e}{m_p}}$$
if the ratio of electron to proton mass, m_e/m_p is not neglected.

29. Consider the effects of the finite size of the proton on the spectrum of the hydrogen atom. Assume that the proton's charge is distributed uniformly inside a small sphere of radius $r_0 = 10^{-13}$ cm and treat $r_0 << \hbar^2/me^2$.

    (a) Use first order perturbation theory to find the resulting shift in the energy of the hydrogen atom s-state with principal quantum number n to lowest non-trivial order in r_0/a_0.

    (b) Show that the energies of states with $l > 0$ are not effected to this order by the proton's finite size.

30. Continue to study problem 30 from Problem Set #6 using the simplified Schrödinger equation:
$$-\frac{d^2}{dx^2} + V(x)\psi(x) = E\psi(x)$$
where $V(x)$ is given by:
$$V(x) = 20e^{-(x-50)^2/48}$$
which describes a barrier penetration problem. Assume that a plane wave with amplitude e^{-ipx} is incident on the barrier from the right, where $p = \sqrt{E}$.

    (a) Find the solution to this equation in the region $0 \le x \le 100$ for E=16 numerically using the WKB approximation. Plot your results for the real and imaginary parts of $\psi(x)$.

(b) Plot the real and imaginary parts of the difference between your answers from parts (a) above and part (a) of problem 20 from Problem Set #6.

(c) Find the WKB result for the magnitude and phase of the transmitted and reflected waves and compare your results with those obtained by numerically integrating the Schrödinger equation in problem 20.

For your solution to this problem you need only submit the two plots obtained from parts (a) and (b) and give the four WKB numbers obtained in part (c). [A possible starting point for this problem might be the Python script WKB.ipynb which is described on the course python webpage.