Physics GR6037 | Professor Christ
due 12/9/2019

Problem Set #11

Don't miss the final two problems on the second page.

36. Consider a potential function $V(x)$ which is everywhere finite and which obeys:

$$V(x) \le 0 \text{ for all } x \quad \text{and} \quad V(x) = 0 \text{ for } |x| > x_0.$$

Show using the variational method that:

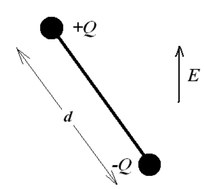
(a) The 1-dimensional potential $\lambda^2 V(x)$ for $-\infty < x < +\infty$ always possesses at least one bound state for arbitrarily small λ.

(b) The 3-dimensional, spherically symmetric potential $\lambda^2 V(|\vec{r}|)$ possesses no bound state if λ is made sufficiently small.

Hint: A good strategy is to compare to the known solutions for the case that $V(x)$ or $V(|\vec{r}|)$ is an appropriately chosen square well potential.

37. Let E_l be the lowest energy eigenvalue for all normalizable eigenstates with orbital angular momentum l, in a spherically symmetric potential $V(|\vec{r}|)$. Prove that if $l > l'$ then $E_l > E_{l'}$.

38. A rod of length d with point charges $+Q$ and $-Q$ at either end is constrained to rotate in the $x-y$ plane about its center. In addition there is a uniform electric field $\vec{E}$ parallel to the y-direction and the system has a moment of inertia I about its center.



(a) What is the Hamiltonian for the system?

(b) If the interaction with the electric field is treated as a perturbation what are the zeroth order energies?

(c) The first order energies

(d) The second order energies?

39. Consider the 2-dimension harmonic oscillator with Hamiltonian

$$H = \frac{p_x^2 + p_y^2}{2m} + \frac{1}{2} m\omega^2 (x^2 + y^2).$$

(a) Determine the energy eigenvalues and corresponding energy eigenstates.

(b) If an extra perturbing term, λxy is added to the energy, use degenerate perturbation theory to find the first-order shift in the energies of the six lowest energy states.

(c) Solve the problem exactly for this case.

(d) If instead an extra perturbing term, $\lambda x^2 y^2$ is added to the energy, use degenerate perturbation theory to again find the first-order shift in the energies of the six lowest states.

40. Find the Clebsch-Gordon coefficients $(j, m|j_1, m_1; j_2, m_2)$ for

   (a) $j_1 = 1/2$, $j_2 = 1$

   (b) $j_1 = 1$, $j_2 = 1$

   (c) $j_1 = 1/2$, $j_2 =$ arbitrary.

41. Two spin-1 particles interact through the Hamiltonian

$$H = g\vec{S}_1 \cdot \vec{S}_2$$

   (a) What are the allowed energies and corresponding eigenstates for this Hamiltonian? Express the eigenstates in terms of the standard basis of states with definite values of $(S_1)_z$ and $(S_2)_z$.

   (b) If initially the system is in the state with $(S_1)_z = (S_2)_z = 0$, what is the probability of finding $(S_1)_z = \hbar$ at a later time t?