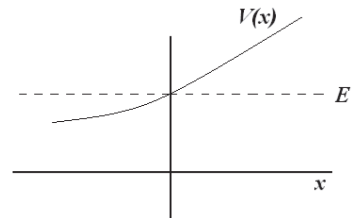
Physics G6037 Professor Christ

due 10/18/2019

Problem Set #6

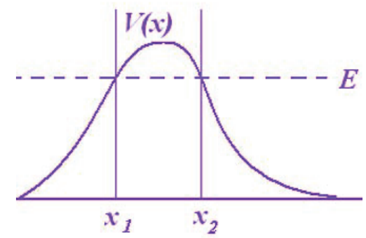
19. A particle of mass m and energy E moves in one dimension under the influence of the potential $V(x)$. If $V(0) = E$ and the particle's wavefunction has the approximate form:



$$\psi(x) = \frac{C}{[E - V(x)]^{\frac{1}{4}}} \cos\left\{\frac{1}{\hbar}\int_x^0 \sqrt{2m(E - V(x'))}dx' + \frac{\pi}{4}\right\}$$

for x large and negative. Find $\psi(x)$ for x large and positive assuming the WKB approximation to be valid both for x sufficiently small to permit a first order Taylor aproximation to $V(x)$ about $x = 0$.

20. A particle with energy E and mass m is incident from the left on the potential barrier $V(x)$.



    (a) What conditions as $x \to \pm\infty$ should be imposed on the energy eigenfunction $\psi_E(x)$ appropriate to this problem?

    (b) Use the WKB approximation to obtain an expression for the probability of finding the particle to the right of the barrier at a much later time t for the case that E is less than the maximum value of V. Be careful to use only the information that is available from the known asymptotic form of the solution, avoiding the addition of elements of the solution which are not known without a more complete solution.

21. Consider the simplified Schrödinger equation:

$$-\frac{d^2}{dx^2} + V(x)\psi(x) = E\psi(x)$$

where $V(x)$ is given by:

$$V(x) = 20e^{-(x-50)^2/48}$$

which describes a barrier penetration problem. Assume that a plane wave with amplitude e^{-ipx} is incident on the barrier from the right, where $p = \sqrt{E}$.

    (a) Find the solution to this equation in the region $0 \le x \le 100$ for E=16 numerically. Plot your results for the real and imaginary parts of $\psi(x)$.

    (b) Find numerical values for the magnitude and phase of the reflected and transmitted waves. Use a step size $Dx = 0.005$ and the leap-frog method implemented in the Schrodinger.ipynb example posted on the web.

For your solution to this problem you need submit only the plot obtained from part a) and the four numbers obtained in part b). [It may be helpful to begin with a working Jupyter notebook which can be downloaded from the course web site, Python, which solves the case $V(x) = 0$ and then to modify this sample program to solve the problem here.]