Physics G6037 Professor Christ

due 11/9/2018

Problem Set #8

*Don't miss the final problem on the second page*

26. Consider the 3-dimensional harmonic oscillator with

$$H = \frac{\vec{p}^{\,2}}{2m} + \frac{1}{2}m\omega^2\vec{r}^{\,2}.$$

(a) Find the allowed energies and corresponding multiplicities for this Hamiltonian.

(b) Observe that the eigenstates of $H$ can be chosen as eigenstates of $L^2$ and $L_z$ as well. What values of $\ell$ and $m$, with what multiplicities are possible for states with a given energy eigenvalue $E$?
[Hint: this can be answered using the your solution to problem 25 in last week's assignment. This might be done by extracting from the product of hermite polynomials $h_{n_1}(x_1)h_{n_2}(x_2)h_{n_3}(x_3)$ in each energy eigenstate that term which contain the largest power of each of its three arguments. Such a relation would provide a 1:1 connection between the homogeneous polynomials of problem 25 and the energy eigenstates of this problem.]

(c) Determine the form of the radial wave function for those states with $E = \hbar\omega(N + \frac{3}{2})$ and $\ell = N$.

27. A spin-1/2 particle with magnetic moment $\vec{\mu} = \gamma\vec{s}$ is placed in a magnetic field $\vec{B}_0 = B_0\hat{z}$ in the $z$-direction. Use eigenstates $| \pm \frac{1}{2}\rangle$ of $s_z$ with eigenvalues $\pm\hbar/2$.

(a) For an initial state $|\psi(0)\rangle = a_+| + \frac{1}{2}\rangle + a_-| - \frac{1}{2}\rangle$, find the resulting state $|\psi(t)\rangle$ at a later time $t$.

(b) Next a time-dependent magnetic field

$$\vec{B}_1(t) = B_1\hat{x}\cos(\omega t) - B_1\hat{y}\sin(\omega t) \tag{1}$$

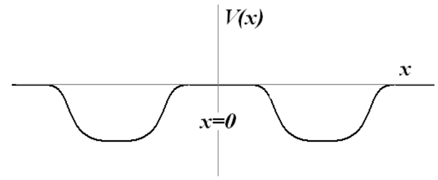
is added to the original field $\vec{B}_0$. Solve the resulting time-dependent Schrödinger equation for these combined magnetic fields by working in a "rotating" frame: deriving and solving the Schrödinger equation obeyed by the transformed state:

$$|\psi\,'(t)\rangle = e^{+i\omega t\hat{z}\cdot\vec{s}/\hbar}|\psi(t)\rangle. \tag{2}$$

[Note: while it offers insight to view this as a transformation to a rotating frame, Eq. (2) can be viewed as a simple mathematical transformation which makes the problem easy to solve without any need for physical intuition.]

(c) Describe the resulting motion of the particle's spin as $\omega$ is varied through the Larmor precession frequency $\omega_L = \gamma B_0$.

28. Consider a one-dimensional potential $V(x)$ of the form shown, symmetric about $x = 0$: $V(x) = V(-x)$.




(a) Prove that the eigenstates of the Hamiltonian

$$H = \frac{\partial^2}{\partial x^2} \frac{\hbar^2}{2m} + V(x)$$

can be chosen to obey $\psi(x) = \pm\psi(-x)$.

(b) If the two potential wells are far separated then the normalizable eigenstates are approximately the superpositions:

$$\psi_{n,\pm}(x) = \phi_n^L(x) \pm \phi_n^R(x)$$

where $\phi_n^L(x) = \phi_n^R(-x)$ is a normalizable eigenfunction for the potential

$$V'(x) = \begin{cases} V(x) & x < 0 \\ 0 & 0 < x. \end{cases}$$

Use the WKB approximation to find the difference in energy of $\psi_{n,+}$ and $\psi_{n,-}$ for the case of a large but finite separation between the wells.

(c) Does the symmetric or anti-symmetric solution have the lower energy?

[Hint: This problem is less complex than the $\alpha$ decay problem worked in class but does require a careful setup to control the ambiguities associated with the WKB approximation. You might work in the region $x > 0$ and identify two solution with exponentially growing or falling behavior to the left of the smallest $x > 0$ turning point. You might then write the normalizable solution $\psi_{E,\pm}(x)$ as a combination of these two states based on their behavior in the classically allowed region for $x > 0$.]