

Problem Set #10

32. Consider the effects of the finite size of the proton on the spectrum of the hydrogen atom. Assume that the proton's charge is distributed uniformly inside a small sphere of radius $r_0 = 10^{-13}$ cm and treat $r_0 \ll \hbar^2/me^2$.
- (a) Use first order perturbation theory to find the resulting shift in the energy of the hydrogen atom s-state with principal quantum number n to lowest non-trivial order in r_0/a_0 .
 - (b) Show that the energies of states with $l > 0$ are not effected to this order by the proton's finite size.
33. Find the first and second order effects of the perturbing potential $V(x) = \lambda x$ on the spectrum of the one-dimensional harmonic oscillator with the unperturbed Hamiltonian

$$H_0 = \frac{p^2}{2m} + \frac{1}{2}m\omega^2 x^2$$

using the standard formalism of perturbation theory. Show that your results agree with the exact solution to the perturbed problem obtained by using a shifted position variable.

34. Consider the Hamiltonian $H = H_0 + \lambda V$ and assume that the spectrum of H_0 is non-degenerate.
- (a) Use Rayleigh-Schrödinger perturbation theory to determine the eigenvalues of H to third order in λ in terms of $E_n^{(0)}$ and $V_{n',n}$ where

$$H_0|\phi_n\rangle = E_n^{(0)}|\phi_n\rangle, \quad \langle\phi_{n'}|V|\phi_n\rangle = V_{n',n}$$
 - (b) Show that your result is consistent with what you would obtain using Wigner-Brillouin perturbation theory.
35. A particle of mass m moves on the surface of a sphere of radius R under the influence of a uniform gravitational field near the Earth's surface. Assume that the sphere is sufficiently small that $\lambda = m^2 g R^3 / \hbar^2$ can be treated as a small expansion parameter.
- (a) What are the two lowest eigenvalues and the corresponding eigenfunctions to zeroth order in λ ?
 - (b) What are the corrections to each of these two eigenvalues to first order in λ ?
 - (c) What are the corrections to second order in λ ? (Note: rules for decomposing the products of spherical harmonics limit the possible intermediate states.)