Physics GR6037 Professor Christ

due 11/23/2025

Problem Set #10

36. A particle of mass m moves on the surface of a sphere of radius R under the influence of a uniform gravitational field near the Earth's surface. Assume that the sphere is sufficiently small that $\lambda = m^2 g R^3/\hbar^2$ can be treated as a small expansion parameter.

    (a) What are the two lowest eigenvalues and the corresponding eigenfunctions to zeroth order in λ?

    (b) What are the corrections to each of these two eigenvalues to first order in λ?

    (c) What are the corrections to second order in λ? (Note: rules for decomposing the products of spherical harmonics limit the possible intermediate states.)

37. Consider a potential function $V(x)$ which is everywhere finite and which obeys:

    $$V(x) \leq 0 \text{ for all } x \text{ and } V(x) < 0 \text{ for some } x \text{ and } V(x) = 0 \text{ for } |x| > x_0.$$

    Show using the variational method that:

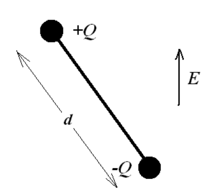
    (a) The 1-dimensional potential $\lambda^2 V(x)$ for $-\infty < x < +\infty$ always possesses at least one bound state for arbitrarily small λ.

    (b) The 3-dimensional, spherically symmetric potential $\lambda^2 V(|\vec{r}|)$ possesses no bound state if λ is made sufficiently small.

    Hint: A good strategy is to compare to the known solutions for the case that $V(x)$ or $V(|\vec{r}|)$ is an appropriately chosen square well potential.

38. Let E_l be the lowest energy eigenvalue for all normalizable eigenstates with orbital angular momentum l, in a spherically symmetric potential $V(|\vec{r}|)$. Prove that if $l > l'$ then $E_l > E_{l'}$.

39. A rod of length d with point charges $+Q$ and $-Q$ at either end is constrained to rotate in the $x-y$ plane about its center. In addition there is a uniform electric field $\vec{E}$ parallel to the y-direction and the system has a moment of inertia I about its center. [Be careful of degeneracy.]



    (a) What is the Hamiltonian for the system?

    (b) If the interaction with the electric field is treated as a perturbation what are the zeroth order energies?

    (c) The first order energies

    (d) The second order energies?

40. Consider the 2-dimension harmonic oscillator with Hamiltonian

    $$H = \frac{p_x^2 + p_y^2}{2m} + \frac{1}{2} m\omega^2 (x^2 + y^2).$$

(a) Determine the energy eigenvalues and corresponding energy eigenstates.

(b) If an extra perturbing term, λxy is added to the energy, use degenerate perturbation theory to find the first-order shift in the energies of the six lowest energy states.

(c) Solve the problem exactly for this case.

(d) If instead an extra perturbing term, $\lambda x^2 y^2$ is added to the energy, use degenerate perturbation theory to again find the first-order shift in the energies of the six lowest states.