Physics GR6037 Professor Christ

due 10/12/2025

Problem Set #5

[Don't miss Problem 21 on the second page.]

18. A particle of mass $m$ moves in one dimension under the influence of the potential $\frac{1}{2}m\omega^2 x_{op}^2$ where $x_{op}$ is the particle's position operator.

    (a) Show that the operator $\exp\{i\Delta p\, x_{op}/\hbar\}$ increases the particle's momentum by the amount $\Delta p$.

    (b) The particle, initially in the ground state, undergoes a sudden collision with a second particle and its momentum is increased by $\Delta p$. Assuming that the effect of this collision can be described by an application of the operator $\exp\{i\Delta p\, x_{op}/\hbar\}$, what is the probability that the particle remains in the ground state?

    (c) What is the probability that it is raised to the $n^{th}$ excited state?

19. A charged, spinless particle moves in a uniform magnetic field $\vec{B}$ pointing in the $z$-direction. Use the gauge choice $A_x = A_z = 0$, $A_y = |\vec{B}|x$.

    (a) Find the allowed energies and corresponding eigenstates for this system. Note, these energy eigenstates can be chosen to also be eigenstates of $p_y$.

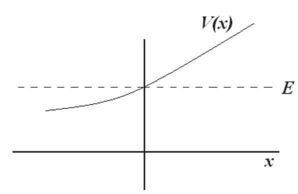
    (b) Use your results to construct a time-dependent state which solves the Schrodinger equation and for which the expectation values of the position operators $x_{\text{op}}$ and $y_{\text{op}}$ approximate the usual, classical circular motion. One possible approach:

        i. Create a SHO coherent state with fixed $p_y$.

        ii. Take a Gaussian superposition of states with different $p_y$.

        iii. Find the location, $\big(x(t), y(t)\big)$, where the wave function is largest as a function of time using the stationary phase approximation.

20. A particle of mass $m$ and energy $E$ moves in one dimension under the influence of the potential $V(x)$. If $V(0) = E$ and the particle's wavefunction has the approximate form:




$$\psi(x) = \frac{C}{[E - V(x)]^{\frac{1}{4}}} \cos\left\{\frac{1}{\hbar}\int_x^0 \sqrt{2m(E - V(x'))}dx' + \frac{\pi}{4}\right\}$$

for $x$ large and negative. Find $\psi(x)$ for $x$ large and positive assuming the WKB approximation to be remain valid when $x$ is sufficiently small to permit a first order Taylor aproximation to $V(x)$ about $x = 0$.

21. Explore the stationary phase approximation by studying a simple numerical example. Consider the following function of the paramters $x$ and $t$:

$$F(x,t) = \int_{-\infty}^{\infty} dk\, e^{-\frac{(k-k_0)^2}{\Delta k^2}} e^{\frac{i}{\lambda}\left(\sin kx - k^2 t\right)}. \tag{1}$$

For a small value $\Delta k$ the integral over $k$ in Eq. (1) will be sharply peaked about $k = k_0$. Likewise when the paramter $\lambda$ is small the value of $x$ at which the integral is largest might be determined by the stationary phase approximation.

(a) What is the relation between $x$ and $t$ implied by the stationary phase approximation?

(b) Write a Python code to perform the integral over $k$ and demonstrate that for $\Delta k = 0.01$, $k_0 = 1.0$, $\lambda = 0.01$, and $t = 1$ the stationary phase approximation works well. For the purpose of this homework you need only include a single graph of $F(x,t)$ for $t =$ and $0 \leq x \leq 10$ as the solution to this part. Instead of solving the equation derived in part 21a to determine the prediction of the stationary phase approximation, you should plot the left- and right-hand sides of that equation as two additional curves and show that $F(x,t)$ is indeed largest for those values of $x$ where these two curves cross. You should rescale the values for each of these two curves by a common factor so that they appear in your plot without changing the scale at which $F(x,t)$ would be plotted alone.
[Hint: you might begin with the sample Python code StationaryPhase and modify it to answer this question. See also more detailed instructions under the Python link, also available on the course webpage.