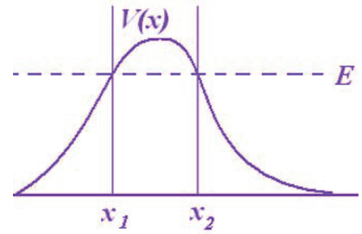
Physics GR6037 | Professor Christ
due 10/26/2025

Problem Set #6



22. A particle with energy E and mass m is incident from the left on the potential barrier $V(x)$.
    (a) What conditions as $x \to \pm\infty$ should be imposed on the energy eigenfunction $\psi_E(x)$ appropriate to this problem?
    (b) Use the WKB approximation to obtain an expression for the probability of finding the particle to the right of the barrier at a much later time t for the case that E is less than the maximum value of V. Be careful to use only the information that is available from the known asymptotic form of the solution, avoiding the addition of elements of the solution which are not known without a more complete solution.

23. Find the three matrices $(J_i)_{mm'} = \langle j, m|J_i|j, m'\rangle$, $1 \le i \le 3$, explicitly for the case $j = 1$. Verify the commutation relation: $[J_1, J_2] = i\hbar J_3$.

24. Consider a quantum mechanical system with total angular momentum $j = \frac{1}{2}$ (for example a proton at rest with intrinsic spin $j = \frac{1}{2}$). Any state $|\psi\rangle$ of the system can be written as a superposition of the two states $|j = \frac{1}{2}, m = \pm\frac{1}{2}\rangle$:

$$|\psi\rangle = a_+|\frac{1}{2}, \frac{1}{2}\rangle + a_-|\frac{1}{2}, -\frac{1}{2}\rangle.$$

where a_+ and a_- are complex numbers.

    (a) If the system is known to be in the state with $J_3 = +\frac{\hbar}{2}$ what is the probability that a measurement of $J_3\cos(\theta) + J_2\sin(\theta)$ will yield the value $+\frac{\hbar}{2}$?
    (b) If $\hat{n}$ is a unit vector pointing in a direction specified by the polar coordinates θ and ϕ find values for the two coefficients a_+ and a_- which determine a state $|\psi(\theta, \phi)\rangle$ which obeys

$$\langle\psi(\theta, \phi)|\vec{J}|\psi(\theta, \phi)\rangle = \frac{\hbar}{2}\hat{n}.$$

    (c) If the time development of this spin-$\frac{1}{2}$ system is determined by the Hamiltonian

$$H = -\frac{ge}{2mc}\vec{J}\cdot\vec{B}$$

    (for example our proton sits in a magnetic field $\vec{B}$), and it is known that at $t = 0$, $J_3 = +\frac{\hbar}{2}$, find the resulting state $|\psi(t)\rangle$ at a later time t. (Note, $\vec{B}$ should be assumed to point in a general direction.)
    (d) For the case that $\vec{B} = |\vec{B}|\hat{z}$ and the initial state is the state $|\psi(\theta, \phi)\rangle$ found in part (b), find the resulting state at a later time t and show that the expectation value $\langle\psi(t)|\vec{J}|\psi(t)\rangle$ has the same time dependence as a classical magnetic dipole with angular momentum $\vec{J}$ and magnetic moment $\vec{\mu} = \frac{ge}{2mc}\vec{J}$ precessing in the magnetic field $\vec{B}$.