Physics GR6037 Professor Christ
due 11/09/2025

Problem Set #8

29. A spin-1/2 particle with magnetic moment $\vec{\mu} = \gamma \vec{s}$ is placed in a magnetic field $\vec{B}_0 = B_0 \hat{z}$ in the z-direction. Use eigenstates $|\pm \frac{1}{2}\rangle$ of s_z with eigenvalues $\pm\hbar/2$.

    (a) For an initial state $|\psi(0)\rangle = a_+|+\frac{1}{2}\rangle + a_-|-\frac{1}{2}\rangle$, find the resulting state $|\psi(t)\rangle$ at a later time t.

    (b) Next a time-dependent magnetic field

    $$\vec{B}_1(t) = B_1 \hat{x}\cos(\omega t) - B_1 \hat{y}\sin(\omega t) \tag{1}$$

    is added to the original field $\vec{B}_0$. Solve the resulting time-dependent Schrödinger equation for these combined magnetic fields by working in a "rotating" frame, deriving and solving the Schrödinger equation obeyed by the transformed state:

    $$|\psi'(t)\rangle = e^{-i\omega t \hat{z}\cdot\vec{s}/\hbar}|\psi(t)\rangle. \tag{2}$$

    [Note: while it offers insight to view this as a transformation to a rotating frame, Eq. (2) can be viewed as a simple mathematical transformation which makes the problem easy to solve without any need for physical intuition.]

    (c) Describe the resulting motion of the particle's spin as ω is varied through the Larmor precession frequency $\omega_L = \gamma B_0$.

30. Consider the Hydrogen atom as a two-particle problem which depends on the two, 3-dimensional coordinates $\vec{r}_e$ and $\vec{r}_p$, the locations of the electron and proton.

    (a) Write down the quantum mechanical Hamiltonian in terms of the positions $\vec{r}_e$ and $\vec{r}_p$ and their conjugate momentum operators $\vec{p}_e$ and $\vec{p}_p$.

    (b) Working with the resulting time-independent Schrödinger equation in position space, change from the coordiates $\vec{r}_e$ and $\vec{r}_p$ to the relative and center of mass coordinates

    $$\vec{r} = \vec{r}_e - \vec{r}_p, \quad \vec{R} = \frac{1}{m_e + m_p}\Big\{m_e\vec{r}_e + m_p\vec{r}_p\Big\}$$

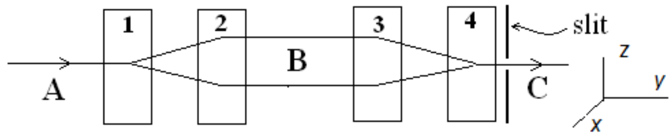
    and rewrite the stationary state Schrödinger equation in terms of these variables. Here m_e and m_p are the masses of the electron and proton.

    (c) Show that the allowed energies of the hydrogen atom become:

    $$E_n = -\frac{m_e e^4}{2\hbar^2}\frac{1}{n^2}\frac{1}{1+\frac{m_e}{m_p}}$$

    if the ratio of electron to proton mass, m_e/m_p is not neglected.

31. A beam of neutral atoms with angular momentum $s = 1/2$ and magnetic moment $\vec{\mu} = \gamma\vec{s}$ enters the Stern-Gerlach style apparatus shown below in region A and exits in region C, moving from left to right. Magnets 1 and 2 produce inhomogenous magnetic fields which point in the z-direction and vary with z. They separate the beam into two parts as shown. The paths of atoms with $s_z = +\hbar/2$ are shifted upward while those with $s_z = -\hbar/2$ are shifted downward. Magnets 3 and 4 provide exactly the opposite fields and bring the two $s_z = \pm\hbar/2$ components coherently back together so that the apparatus has ultimately identical affects on both the $s_z = \pm\hbar/2$ states.



(a) If the incoming atoms are prepared in the state with $s_x = +\hbar/2$ what is the probability that a given atom will be found on the upper path in the figure? In the lower path?

(b) An additional magnetic field $\vec{B}(y)$ in a fixed direction but with a magnitude which varies with position is introduced in region B which affects only atoms moving on the upper path in that region. The total, time-integrated magnetic field seen by those atoms which traverse the upper path is $\vec{I}_B = \int \vec{B}(y(t))dt$. If $\vec{I}_B$ points in the z direction, find the polarization of the atoms seen in region C as a function of I_B. Recall that the polarization is the three-dimensional vector whose components are the average values of the three spin operators s_x, s_y, s_z measured independently on many identically prepared copies of the same state. (Be sure to combine the contributions from both the upper and lower paths.) [Note: the change in phase of the state which traverses the upper path, which has $J_z = +\hbar/2$, is directly related to the time integrated quantity $\vec{I}_B$.]

(c) For a situation identical to (b) but with $\vec{I}_B$ oriented in the y-direction, what is the probability that an atom entering from the left with $s_x = +\hbar/2$ will finally appear in region C? (Note the slit prevents atoms that are not deflected downward by magnets 3 and 4 from reaching region C.)

(d) For the arrangement in (c), what will be the polarization of those atoms which are found in region C?

(e) Sufficient photographic emulsion is placed in the lower path in region B so that an atom passing through that lower path will cause at least one film grain to be exposed. Assume that the emulsion has no effect on the atoms passing through it. Find the average polarization measured for the atoms found in region C for the cases (b) and (c) above.

Note: this is an idealized problem in which the motion of the atom is specified and is not directly affected by the magnetic field imposed in region B. Only the time dependence of the atom's spin and the effect of the resulting spin on the atom's motion as it passes through magnets 3 and 4 needs to be considered when solving this problem.