Physics GR6037 Professor Christ

due 11/16/2025

Problem Set #9

32. Continue to study problem 25 from Problem Set #7 using the simplified Schrödinger equation:

$$-\frac{d^2}{dx^2} + V(x)\psi(x) = E\psi(x)$$

where $V(x)$ is given by:

$$V(x) = 20e^{-(x-50)^2/48}$$

which describes a barrier penetration problem. Assume that a plane wave with amplitude e^{-ipx} is incident on the barrier from the right, where $p = \sqrt{E}$.

(a) Find the solution to this equation in the region $0 \leq x \leq 100$ for E=16 numerically using the WKB approximation. Plot your results for the real and imaginary parts of $\psi(x)$.

(b) Plot the real and imaginary parts of the difference between your answers from parts (a) above and part (a) of problem 25 from Problem Set #7.

(c) Find the WKB result for the magnitude and phase of the transmitted and reflected waves and compare your results with those obtained by numerically integrating the Schrödinger equation in problem 25.

For your solution to this problem you need only submit the two plots obtained from parts (a) and (b) and give the four WKB numbers obtained in part (c). [A possible starting point for this problem might be the Python script WKB.ipynb which is described on the course python webpage.

33. Consider the effects of the finite size of the proton on the spectrum of the hydrogen atom. Assume that the proton's charge is distributed uniformly inside a small sphere of radius $r_0 = 10^{-13}$ cm and treat $r_0 << \hbar^2/me^2$.

(a) Use first order perturbation theory to find the resulting shift in the energy of the hydrogen atom s-state with principal quantum number n to lowest non-trivial order in r_0/a_0.

(b) Show that the energies of states with $l > 0$ are not effected to this order by the proton's finite size.

34. Find the first and second order effects of the perturbing potential $V(x) = \lambda x$ on the spectrum of the one-dimensional harmonic oscillator with the unperturbed Hamiltonian

$$H_0 = \frac{p^2}{2m} + \frac{1}{2}m\omega^2 x^2$$

using the standard formalism of perturbation theory. Show that your results agree with the exact solution to the perturbed problem obtained by using a shifted position variable.

35. Consider the Hamiltonian $H = H_0 + \lambda V$ and assume that the spectrum of H_0 is non-degenerate.

(a) Use Rayleigh-Schrödinger perturbation theory to determine the eigenvalues of H to third order in λ in terms of $E_n^{(0)}$ and $V_{n',n}$ where

$$H_0|\phi_n\rangle = E_n^{(0)}|\phi_n\rangle, \quad \langle\phi_{n'}|V|\phi_n\rangle = V_{n',n}$$

(b) Show that your result is consistent with what you would obtain using Wigner-Brillouin perturbation theory.