Physics GR6037 Professor Christ
Final Examination (practice) 12/09/2025

Answer each of the following **three (3)** questions. Please give a complete description of your method of solution since *partial credit* will be given.

1. Two particles, each with mass m and positions $\vec{r}_1$ and $\vec{r}_2$ move in a 3-dimensional harmonic oscillator potential $V(\vec{r}_1, \vec{r}_2) = \frac{1}{2}m\omega^2({\vec{r}_1}^{\,2} + {\vec{r}_2}^{\,2})$.

   (a) Write down the allowed energies and describe the corresponding eigenstates of this system. [5 points]

   (b) If each particle carries a charge q and they interact through the Coulomb potential $q^2/|\vec{r}_1 - \vec{r}_2|$ find the shift in the ground state energy caused by this electrostatic repulsion to order q^2. (The formula $\int_{-\infty}^{\infty} dz\, e^{-z^2} = \sqrt{\pi}$ may be useful when evaluating your result.) [16 points]

   (c) Improve on your result from part (b) by using the variational method with the trial wave function:
   $$Ne^{-\alpha\frac{m\omega}{2\hbar}({\vec{r}_1}^{\,2}+{\vec{r}_2}^{\,2})}.$$
   Find the equation which determines the parameter α and solve it to first order in q^2. Compare the resulting variational energy evaluated to order q^4 with the first-order energy obtained in part (b). [12 points]

2. A particle of mass m moving in a square, 2-dimensional box of side L obeys periodic boundary conditions at the walls. A small perturbing potential energy $V(x, y) = \lambda \sin[2\pi(x-y)/L)]$ also affects the particle.

   (a) Write down the energy eigenvalues and eigenstates for this problem to zeroth order in λ. [5 points]

   (b) Find the changes in the eigenvalues through first order in λ and their eigenstates to zeroth order in λ for the five states with the lowest unperturbed energies. (Be careful of degeneracies.) [15 points]

   (c) Find all eigenvalues through first order in λ and their corresonding eigenstates through zeroth order in λ. [14 points]

3. Consider a deuterium atom in the electronic state with $j = 1/2$, $n = 1$ and $l = 0$. The deuterium nucleus has spin I=1 and the interaction between this nuclear spin and the electron spin results in an energy splitting $\hbar\omega_{hfs}$ between the states when the nuclear and electron spins combine to form a state with total nuclear and spin angular momentum of $\frac{3}{2}\hbar$ and $\frac{1}{2}\hbar$. You may express your answers to the three questions below in terms of ω_{hfs}. (You should calculate any needed Clebsch-Gordan coefficients.)

   (a) If at $t = 0$ the deuteron spin and the electron spin are both aligned with the positive z direction ($m_I = 1$, $m_s = 1/2$) find the expectation value of the deuteron spin $\vec{I}$ at a later time t. [5 points]

   (b) If at $t = 0$, $m_I = +1$ but $m_s = -1/2$ find the expectation value of the deuteron spin $\vec{I}$ at a later time t. [10 points]

   (c) Find the time average of the expectation values obtained in a) and b). If at $t = 0$ the deuteron spins in a sample of many deuterium atoms are all aligned in the positive z direction, but the electron spins are randomly oriented, what will be the time-averaged deuteron polarization, averaged also over the atoms in the sample? [10 points]