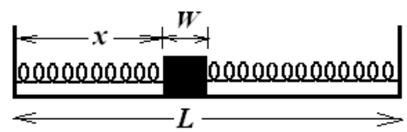
Physics GR6037 Professor Christ
Midterm Examination (practice) 10/15/2025

Answer **two (2)** of the following three questions. Please give a complete description of your method of solution since *partial credit* will be given.

1. A quantum mechanical block of mass M, width W and located by the variable x slides without friction on a horizonal surface. It is attached by two identical massless springs of spring constant k and equilibrium length l to left and right walls separated by a distance L. The block is constrained to move in one dimension parallel to the springs and oscillates about its equilibrium position at the center.

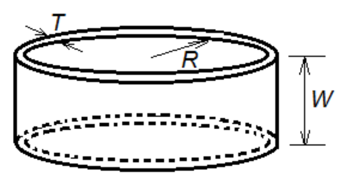


   (a) Write down the quantum Hamiltonian for this system as well as the corresponding energy eigenvalues. What is the wave function $\psi_0(x)$ for the ground state? [15 points]

   (b) At $t = 0$ the right hand the spring is suddenly broken. What is the new Hamiltonian for the system, the new energy eigenvalues and the wave function $\psi'_0(x)$ for the new ground state? [15 points]

   (c) Assume that for $t \leq 0$ the system is in its ground state and that at $t = 0$ the right hand spring is broken so quickly that the quantum state of the system remains unchanged after the spring is broken. Find the probability that the block will be found in the ground state of the new Hamiltonian. [20 points]

2. A particle of mass M is confined by an infinite potential well to move inside a cylindrical shell of inner radius R, thickness T and height W.



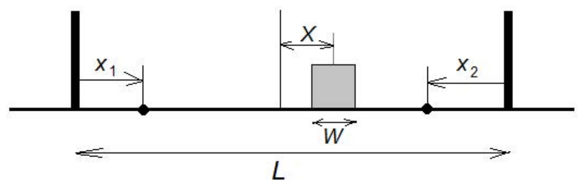
   (a) Introduce cylindrical coordinates and determine the time-independent Schrodinger equation using these coordinates. [10 points]

   (b) Find the energy eigenstates and corresponding eigenvalues for this system. You may assume $T \ll R$ when finding these eigenstates and eigenvalues. [40 points]

3. Consider a particle of mass m and position x moving in one dimension between two walls separated by a distance L. Assume that the particle's wave function is required to vanish at the location of each wall.

   (a) Find the energy eigenstates and eigenvalues for this system. [5 points]

   (b) Use the dependence of the ground state energy on the position of each wall to calculate the force which the particle exerts on each wall when it is in the ground state. [10 points]

   Generalize the situation above to a system of two particles of mass m, one with position x_1 to the left and the other with position x_2 to the right of a fixed block of mass M and width W that is displaced by a distance X to the right from a position midway between two fixed walls which themselves are separated by a fixed distance L. Each particle's wave function is requred to vanish at the walls and at the surface of the block.

   (c) Find the energy eigenstates and eigenvalues of the system. [10 points]

   (d) Finally, we allow the block to move. If $M \gg m$, assume that the smallest energy eigenvalues of the combined system of the two particles and the block can be obtained by treating the block as moving in an X-dependent potential given by the sum of ground state energies of the two particles found in part (c). Write down the resulting time-independent Schrodinger equation obeyed by the block. [5 points]

   (e) Assuming that $|X| \ll L$, use that Schrodinger equation to find the energy eigenstates and eigenvalues for the block. [20 points]

   [This is a simple model of a molecule where the block M represents an atomic nucleus while the two masses m are analogous to electrons and the approximation being made exploiting $M \gg m$ is that of Born and Oppenheimer.]