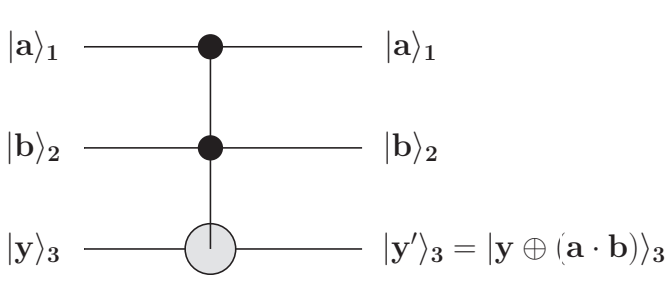
Physics G6038 Professor Christ

due 02/23/2025

Problem Set #15

50. Show that a two-input and two-output quantum gate for which one of the outputs is the AND of its two inputs cannot be realized.

51. Find the 8×8 unitary matrix that describes the action of the Toffoli gate shown in the figure. Assign the number $n = a + 2b + 4y$, $0 \leq n < 8$ to the basis state $|a\rangle_1 \otimes |b\rangle_2 \otimes |y\rangle_3$ and label the columns and rows of the 8×8 matrix to the basis vector associated with the number n,



where n is assigned in ascending order moving from left to right and top to bottom respectively. Here a, b and y take the values 0 and 1. For the labeling the rows, the state $|y\rangle_3$ should be replaced by the state $|y'\rangle_3$. (Here the logical product $a \cdot b$ indicates the AND of a and b.)

52. Design a quantum circuit that will teleport a two-Qbit quantum state (i.e. a general state in the four-dimensional complex vector space of sums of products of pairs of Qbits) between two distant observers: a "two-Qbit teleporter". Prove that your design works.

    How would you generalize your two-Qbit teleporter into an N-Qbit teleporter? How many classical binary bits must be exchanged in your N-Qbit teleporter between the remote systems?

53. The nucleus of a tritium atom ($Z = 1$, $A = 3$) decays into an He^3 nucleus emitting a 17 KeV electron. If the tritium atom was in the ground state, calculate the numerical values of the probabilities that the resulting He^+ ion is found in the $1s$, $2s$ or $2p$ states. Justify any approximation that you make.