Physics GR6038 Professor Christ

due 03/02/2025

Problem Set #16

54. Consider a one-dimensional harmonic oscillator with Hamiltonian

$$H = \frac{p^2}{2m} + \frac{1}{2}m\omega^2(x - x_0)^2$$

If the oscillator is initially in its ground state, find the energy transferred if

(a) x_0 is slowly changed to $x_0' = x_0 + D$.

(b) x_0 is suddenly changed to $x_0' = x_0 + D$.

(c) ω is slowly changed to ω'.

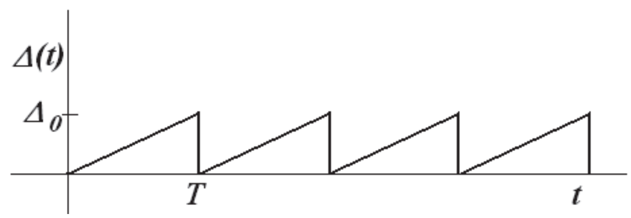
(d) ω is suddenly changed to ω'.

55. Consider a particle moving in a simple harmonic potential in one dimension with the Hamiltonian:

$$H = \frac{p^2}{2m} + \frac{1}{2}m\omega^2\Big(x - \Delta(t)\Big)^2.$$

The displacement $\Delta(t)$ depends on time in the following way:

$$\Delta(t) = \Delta_0\{t/T - \mathrm{int}(t/T)\},$$



where $\mathrm{int}(r)$ denotes the integer part of the real number r. Assume that $m(\Delta_0/T)^2 \ll \hbar\omega$, that the function $\Delta(t)$ returns rapidly to zero when t/T is an integer and that the system is in the ground state at $t = 0$.

(a) What is the state just before $t = T$?

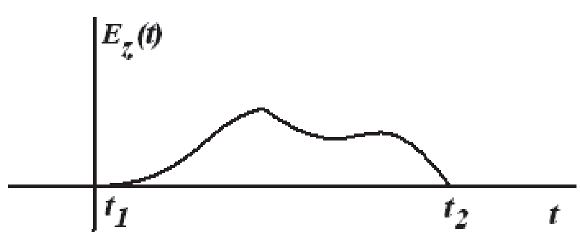
(b) What is the state just after $t = T$?

(c) How much energy has been transferred to the system just after $t = T$?

(d) What is the state just after $t = 2T$ and what is the total energy transfer?

(e) How much energy has been transferred in total to the system just after $t = NT$ where N is an integer and what is the resulting state?

56. Consider a hydrogen atom to which a uniform electric field is slowly applied in the z-direction and slowly reduced again to zero as indicated in the figure. Use the simplest non-relativistic description and ignore electron spin.



(a) If the atom is initially in the ground state, find the probability that it is found in the ground state at a later time $t > t_2$.

(b) If the atom is initially in the $n = 2$, $l = 0$ state, write an expression for the probabilities that at $t > t_2$ it is found in the states with $n = 2$, $l = 1$ and $m_l = +1, 0, -1$. Be careful to properly include the effects of degeneracy.

This problem should be solved using the adiabtic approximation. For part a) perturbation theory should not be needed. However, for part b) you may use first order perturbation theory for the linear Stark effect energy shifts to express the transition probabilities in terms of a time integral of the applied electric field, $E_z(t)$.

57. A particle with spin $\vec{J}$ and magnetic moment $\vec{\mu} = \gamma\vec{J}$ is placed in a time-varying magnetic field $\vec{B}(t)$. At $t = 0$ the field points in the z-direction, $\vec{B}(t = 0) = B\hat{z}$. As time evolves, $\vec{B}(t)$ rotates around an axis in the $x - z$ plane with angular velocity

$$\vec{\omega}_0 = \omega_0\Big(\cos(\phi_0)\hat{z} + \sin(\phi_0)\hat{x}\Big). \tag{1}$$

Assume that this angular rotation frequency ω_0 is much smaller than the Larmor precession frequency $\omega_{\text{prec}} = \gamma B$: $\omega_0 \ll \omega_{\text{prec}}$. Further, assume that at time $t = 0$ the particle is in the state $|\psi(t = 0)\rangle = |j, m\rangle$. Here $\vec{J}^2 = \hbar^2 j(j + 1)$ and $\{|j, m\rangle\}_{-j\le m\le j}$ are the usual eigenstates of $\vec{J}^2$ and J_z.

(a) Use the adiabatic approximation to determine an expression for the state at the time t: $|\psi(t)\rangle$. This can be a concrete formula that involves the rotation operator:

$$R(\hat{\omega}_0, \theta = \omega_0 t) = e^{-\frac{i}{\hbar}\vec{\omega}_0\cdot\vec{J}t}$$

(b) For a time T such that $\omega_0 T = 2\pi$ show that the state returns to itself:

$$|\psi(T)\rangle = e^{i\phi_B(j,m)}e^{i\omega_{\text{prec}}\, mT}|\psi(0)\rangle$$

and determine the "Berry" phase $\phi_B(j, m)$. Express $\phi_B(j, m)$ in terms of the solid angle subtended by the two-dimensional surface whose boundary is the closed curve traced out by the tip of the time-varying field $\vec{B}(t)$.

Hint: you might base your solution on the time varying eigenstates of $H(t)$:

$$|j, m, t\rangle = e^{-i\vec{\omega}_0\cdot\vec{J}t/\hbar}|j, m\rangle.$$

Since these states will typically not obey the condition,

$$\langle j, m, t|\frac{d}{dt}|j, m, t\rangle = 0$$

required by our derivation of the adiabatic theorem, determine and include an extra time dependent phase factor $e^{i\phi_{j,m}(t)}$ so that this condition is obeyed. If you adopt this approach then $\phi_{j,m}(T)$ enters the determination of $\phi_B(j, m)$.