

## Problem Set #20

68. Find the effective inter-nucleus potential caused by the orbiting electron for the ground state of the  $H_2^+$  molecule, using the Born-Oppenheimer approximation. You may follow the treatment given in Baym, using an electron wave function constructed from a superposition of two hydrogenic wavefunctions, each centered on one of the protons.
69. Two identical spin-1/2 particles of mass  $m$  are confined by an infinite potential to move in a 3-dimensional box of side  $L$ . They interact with a spin-independent potential  $V(\vec{r}, \vec{r}') = \lambda \delta(\vec{r} - \vec{r}')$ .
- (a) Find the three lowest energies of the allowed two-particle states when  $\lambda = 0$ .
  - (b) What are the corresponding eigenstates?
  - (c) Find the corrections to the energies of these eigenstates to first order in  $\lambda$ , being careful to deal correctly with degeneracy.
70. Imagine that the electrons in an atom with nuclear charge  $Z$  are actually bosons with spin zero.
- (a) Find the ground state and ground state energy for the resulting atom, neglecting the electrostatic interaction between the electrons.
  - (b) Calculate the shift in the ground state energy determined in part (a) to first order in the potential describing the electrostatic repulsion between the orbiting electrons.
  - (c) If these bosonic electrons have spin 1, what is the degeneracy of ground state neglecting any magnetic effects associated with the particle spins?
  - (d) Assume these identical, spin-1 particles have magnetic moment  $\vec{\mu} = \gamma \vec{J}$  and that the atom is placed in a uniform magnetic field  $\vec{B}$ . Find the new ground state, its energy and degeneracy, treating the magnetic field as weak. Again neglect any spin-spin interactions between the bosons.
  - (e) For the situation studied in part (d) find the first and second excited states, their energies and degeneracies.