Physics GR6038 Professor Christ
due 04/27/2025

Problem Set #22

The wording of problem 77 has been improved

74. Consider an incoming solution $\psi^{\rm in}_{\vec p}(\vec r)$ to the problem of scattering from a central potential $V(\vec r)$. For large $|\vec r|$ the incoming solution $\psi^{\rm in}_{\vec p}(\vec r)$ has the asymptotic form:

$$\psi^{\rm in}_{\vec p}(\vec r) = e^{i\vec p\cdot\vec r/\hbar} + f(\theta,\phi)\frac{e^{ipr/\hbar}}{r}.$$

Show that the vanishing of the total flux of probability current through the surface of a large sphere centered at the origin, implies the optical theorem:

$$\sigma_{\rm total} = \int d\Omega |f(\theta,\phi)|^2 = \frac{4\pi\hbar}{p}{\rm Im}\left\{f(0,0)\right\}.$$

[Hint: This problem requests a standard text-book derivation which requires that the leading term in an expansion in powers of $1/r$ be evaluated for an integral of the form

$$\begin{aligned}\int_0^2 dz f(z)e^{ikrz} &= \frac{-i}{kr}\int_0^2 f(z)\frac{d}{dz}\left[e^{ikrz}\right]dz \\ &= \frac{-i}{kr}\left[f(2)-f(0)\right] + \frac{i}{kr}\int_0^2 \frac{d}{dz}\left[f(z)\right]e^{ikrz}\end{aligned}$$

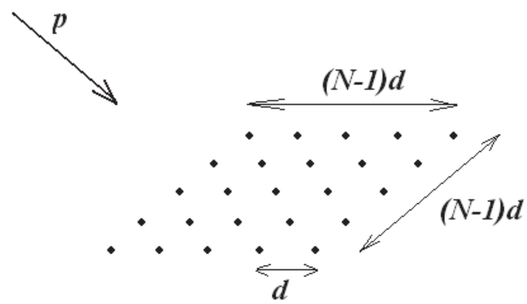
where the second line is obtained by integration by parts and the second term in the second line can be shown to fall as $1/r^2$ for large r by repeating this argument.]

75. Find the Born approximation to the differential cross section $d\sigma/d\Omega$ for the scattering of two spin-1/2 particles with masses m_1 and m_2 and interaction potential

$$V(r) = \frac{\lambda}{|\vec r|^2}\vec\sigma_1\cdot\vec\sigma_2$$

in the center of mass frame. You should assume that the initial particles are not polarized, which requires that you average over all possible initial state polarizations. You should also assume that the polarization of the final particles is not measured so all possible final polarizations must be summed together. Here $\vec r$ is the relative position of particle 1 with respect to particle 2 and σ_1^i and σ_2^i, $i = 1, 2, 3$ are the Pauli matrices acting on the spin indices of particles 1 and 2 respectively. [This problem is likely to be most easily solved using the stationary-state formalism.]

76. Determine the lowest order differential cross section $d\sigma/d\Omega$ for the scattering of electrons with initial momenta $\vec{p} = (p_x, p_y, p_z)$ from a square, two-dimensional lattice of fixed charges q with lattice spacing d lying in the $x - y$ plane.



77. Consider the scattering of two particles with masses m_1 and m_2. If the scattering takes place with a total center of mass energy E where only the $l = 0$ and $l = 1$ and phase shifts δ_0 and δ_1 are non-zero, find the differential cross section for scattering in the center of mass system.