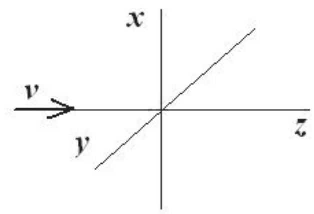
Physics GR6038 Professor Christ
Midterm Examination (practice) 3/25/2025

Answer each of the following **three (3)** questions. (Note, in the real midterm you should expect only two such problems).

1. A neutron with spin $\vec{s}$ and magnetic moment $\vec{\mu} = \frac{ge}{2mc}\vec{s}$ moves slowly along the z axis with velocity v. At $t = 0$ the neutron passes the point $z = 0$ and suddenly enters a region of varying magnetic field:

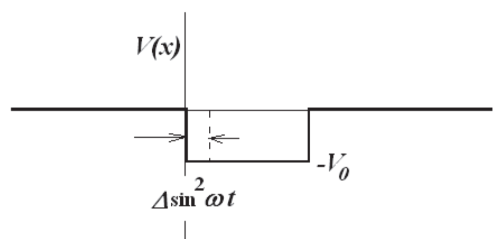
$z < 0$	$z > 0$
$B_x = 0$	$B_x = B\cos z/l$
$B_y = 0$	$B_y = B\sin z/l$
$B_z = 0$	$B_z = 0$



   (a) If the neutron's spin is initially in the $+\hat{x}$ direction before it enters the region of non-zero field, find the neutron's quantum state and spin direction at a later time t. [20 points]

   (b) If the neutron's spin is initially in the $+\hat{z}$ direction before it enters the region of non-zero magnetic field, how far must the neutron travel to have it's spin direction reversed? [13 points]

2. A particle of mass m moving in one dimension is initially in the ground state of a square well of depth $-V_0$ and width L. At $t = 0$ the left wall of the potential begins to vibrate so that:



$$V(x,t) = \begin{cases} 0 & x < \Delta \sin^2(\omega t) \\ -V_0 & \Delta \sin^2(\omega t) \le x \le L \\ 0 & L < x \end{cases}$$

for $t > 0$. The distance Δ can be assumed to be small which allows the use of time-dependent perturbation theory and the simplification of the matrix element which causes the excitation of the initial ground state. In the limit of large time, this time-varying potential will excite the initial state into a positive energy final state, allowing the initially bound particle to escape to infinity.

   (a) Find the excitation rate of the initial state into each possible final state to lowest order in Δ. [23 points]

   (b) What positive energies can be found in the final state at large time? [10 points]

You may assume that $\sqrt{2mV_0} \gg \hbar/L$ to simplify the discussion of the bound state and that $\hbar\omega \gg V_0$ so that V_0 can be ignored when discussing the properties of the final states.

3. Consider a system of N, identical, spinless bosons, moving in a three-dimensional simple harmonic potential $V(\vec{r}) = \frac{1}{2}m\omega^2\vec{r}^{\,2}$.

   (a) What is the ground state, the ground state energy and the ground state degeneracy for the system? [5 points]

   (b) If these bosons interact through a 2-body potential $V(\vec{r}_1, \vec{r}_2) = \lambda\delta(\vec{r}_1 - \vec{r}_2)$, find the shift in this ground state energy to first order in λ. [15 points]

   (c) Repeat your answer to part a) for the case that the bosons have spin 1. [7 points]

   (d) For this spin-1 case, assume that the bosons interact through a 2-body potential $V(\vec{r}_1, \vec{r}_2) = \lambda\vec{s}_1 \cdot \vec{s}_2\delta(\vec{r}_1 - \vec{r}_2)$ where $\vec{s}_i$ is the spin angular momentum operator for the i^{th} boson. If $\lambda < 0$ find the first-order ground state energy and degeneracy. [7 points]

   (e) Find the first-order ground state energy for part d) but with $\lambda > 0$. [0 points]