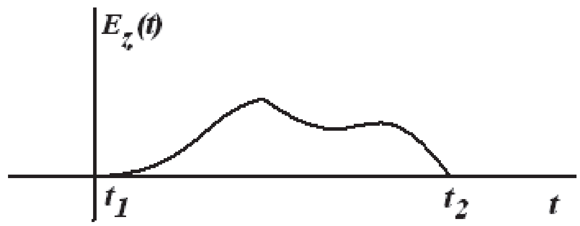
Physics GR6038 — Professor Christ

due 02/22/2026

Problem Set #15

Don't miss problem 60 on the second page.

58. Consider a hydrogen atom to which a uniform electric field is slowly applied in the z-direction and slowly reduced again to zero as indicated in the figure. Use the simplest non-relativistic description and ignore electron spin.



   (a) If the atom is initially in the ground state, find the probability that it is found in the ground state at a later time $t > t_2$.

   (b) If the atom is initially in the $n = 2$, $l = 0$ state, write an expression for the probabilities that at $t > t_2$ it is found in the states with $n = 2$, $l = 1$ and $m_l = +1,\ 0,\ -1$. Be careful to properly include the effects of degeneracy.

   This problem should be solved using the adiabtic approximation. For part a) perturbation theory should not be needed. However, for part b) you may use first order perturbation theory for the linear Stark effect energy shifts to express the transition probabilities in terms of a time integral of the applied electric field, $E_z(t)$.

59. A particle with spin $\vec{J}$ and magnetic moment $\vec{\mu} = \gamma\vec{J}$ is placed in a time-varying magnetic field $\vec{B}(t)$. At $t = 0$ the field points in the z-direction, $\vec{B}(t = 0) = B\hat{z}$. As time evolves, $\vec{B}(t)$ rotates around an axis in the $x - z$ plane with angular velocity

$$\vec{\omega}_0 = \omega_0\Big(\cos(\phi_0)\hat{z} + \sin(\phi_0)\hat{x}\Big). \tag{1}$$

   Assume that this angular rotation frequency ω_0 is much smaller than the Larmor precession frequency $\omega_{\text{prec}} = \gamma B$: $\omega_0 \ll \omega_{\text{prec}}$. Further, assume that at time $t = 0$ the particle is in the state $|\psi(t = 0)\rangle = |j, m\rangle$. Here $\vec{J}^2 = \hbar^2 j(j + 1)$ and $\{|j, m\rangle\}_{-j\le m\le j}$ are the usual eigenstates of $\vec{J}^2$ and J_z.

   (a) Use the adiabatic approximation to determine an expression for the state at the time t: $|\psi(t)\rangle$. This can be a concrete formula that involves the rotation operator:

$$R(\hat{\omega}_0, \theta = \omega_0 t) = e^{-\frac{i}{\hbar}\vec{\omega}_0\cdot\vec{J}t}$$

   (b) For a time T such that $\omega_0 T = 2\pi$ show that the state returns to itself:

$$|\psi(T)\rangle = e^{i\phi_B(j,m)}e^{i\omega_{\text{prec}}\, mT}|\psi(0)\rangle$$

   and determine the "Berry" phase $\phi_B(j, m)$. Express $\phi_B(j, m)$ in terms of the solid angle subtended by the two-dimensional surface whose boundary is the closed curve traced out by the tip of the time-varying field $\vec{B}(t)$.

Hint: you might base your solution on the time varying eigenstates of $H(t)$:

$$|j,m,t\rangle = e^{-i\vec{\omega}_0\cdot\vec{J}t/\hbar}|j,m\rangle.$$

Since these states will typically not obey the condition,

$$\langle j,m,t|\frac{d}{dt}|j,m,t\rangle = 0$$

required by our derivation of the adiabatic theorem, determine and include an extra time dependent phase factor $e^{i\phi_{j,m}(t)}$ so that this condition is obeyed. If you adopt this approach then $\phi_{j,m}(T)$ enters the determination of $\phi_B(j,m)$.

60. Show explicitly that $P_d(t) + \int d^3p\, P_{\vec{p}}(t) = 1$ in the Wigner-Weisskopf description of the decay of a discrete state $|d\rangle$ into a continuum of states $|\phi(\vec{p})\rangle$. The quantity $P_d(t)$ is the probability of finding the system in the state $|d\rangle$ at time t while $P_{\vec{p}}(t)$ is the probability density of finding the decay product in the final state $|\phi(\vec{p})\rangle$, per unit volume in the momentum $\vec{p}$ Assume that at $t=0$, $P_d(0) = 1$ and $P_{\vec{p}}(0) = 0$.