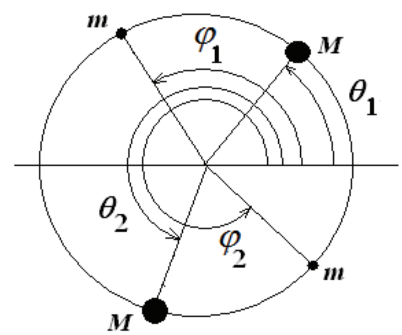
Physics GR6038 Professor Christ
due 04/05/2026

Problem Set #19

71. Two particles of mass m and two particles of mass M move on a circle of radius R. Locate the particles with mass m with the angles, φ_1 and φ_2 and those with mass M with θ_1 and θ_2. The particles interact as hard spheres of radius $r/2$ so the interaction energy between the particles at the angles ϕ_i and θ_j is:



$$V(\theta_i - \theta_j) = \begin{cases} 0 & |\varphi_i - \theta_j| > r/R \\ \infty & |\varphi_i - \theta_j| < r/R. \end{cases}$$

Assume that the particles are arranged as shown in the figure, that $M \gg m$ and neglect r/R.

(a) If the two heavy particles are at fixed locations θ_1 and θ_2, find the ground state energy for the light particles.

(b) Use the Born-Oppenheimer approximation to find the ground state and ground state energy for the entire system.

72. Solve the Thomas-Fermi equation $d^2\phi(x)/dx^2 = \phi^{3/2}/x^{1/2}$ numerically using Python in the region $0 \le x \le 10$. Your solution need not be very accurate for $x > 10$ but should not diverge. See, for example, Harry Krutter, Journal of Computational Physics, 47, 308-3 12 (1982) for some insight into the solutions of this non-linear equation.

(a) Make a plot of your solution.

(b) What value do you find for $\phi(5)$?

(c) Calculate the largest value of $x\phi(x)$ to at least 1% accuracy.

[Suggestions: Rather than using a uniform grid $x_n = n \cdot \Delta x$ to integrate the Thomas-Fermi equation you might experiment with a grid such as $x_n = y_n^2/(1 + y_n)$ with $y_n = n \cdot \Delta y$ chosen to have a greater density of points where the equation is most singular. The initial slope $d\phi(x)/dx|_{x=0}$ might be most easily determined by trial and error.]