Physics GR6038 — Professor Christ

due 04/05/2026

Problem Set #20

73. Find the effective inter-nucleus potential caused by the orbiting electron for the ground state of the $H_2^+$ molecule, using the Born-Oppenheimer approximation. You may follow the treatment given in Baym, using an electron wave function constructed from a superposition of two hydrogenic wavefunctions, each centered on one of the protons.

74. Two identical spin-1/2 particles of mass $m$ are confined by an infinite potential to move in a 3-dimensional box of side $L$. They interact with a spin-independent potential $V(\vec{r}, \vec{r}\,') = \lambda\delta(\vec{r} - \vec{r}\,')$.

    (a) Find the three lowest energies of the allowed two-particle states when $\lambda = 0$.

    (b) What are the corresponding eigenstates?

    (c) Find the corrections to the energies of these eigenstates to first order in $\lambda$, being careful to deal correctly with degeneracy.

75. Imagine that the electrons in an atom with nuclear charge $Z$ are actually bosons with spin zero.

    (a) Find the ground state and ground state energy for the resulting atom, neglecting the electrostatic interaction between the electrons.

    (b) Calculate the shift in the ground state energy determined in part (a) to first order in the potential describing the electrostatic repulsion between the orbiting electrons.

    (c) If these bosonic electrons have spin 1, what is the degeneracy of ground state neglecting any magnetic effects associated with the particle spins?

    (d) Assume these identical, spin-1 particles have magnetic moment $\vec{\mu} = \gamma\vec{J}$ and that the atom is placed in a uniform magnetic field $\vec{B}$. Find the new ground state, its energy and degeneracy, treating the magnetic field as weak. Again neglect any spin-spin interactions between the bosons.

    (e) For the situation studied in part (d) find the first and second excited states, their energies and degeneracies.