Physics GR6038 | Professor Christ
due 04/12/2026

Problem Set #21

76. Using the Schrodinger equation obeyed by the density matrix for the damped simple harmonic oscillator:

$$\dot{\rho} = -i\omega_0'[a^\dagger a, \rho] + \frac{\gamma}{2}\left[2a\rho a^\dagger - a^\dagger a\rho - \rho a^\dagger a\right] + \gamma\overline{n}\left[a\rho a^\dagger + a^\dagger\rho a - a^\dagger a\rho - \rho a a^\dagger\right], \quad (1)$$

show that the usual Boltzmann distribution:

$$\rho = \frac{e^{-\beta\hbar\omega a^\dagger a}}{\mathrm{Tr}\left\{e^{-\beta\hbar\omega a^\dagger a}\right\}}$$

is a time-independent solution provided β is chosen so that $\mathrm{Tr}\left\{a^\dagger a\rho\right\} = \overline{n}$.

77. Again consider the simple harmonic oscillator introduced in problem 76.

(a) If at $t = 0$ the density matrix is diagonal in the basis of energy eigenstates:

$$\rho = \sum_n p_n(0)|n\rangle\langle n| \quad (2)$$

show that it will remain diagonal at later times.

(b) Find the coupled differential equations obeyed by the probabilities $p_n(t)$. Note the validity of these equations for the diagonal elements of ρ when expressed in the basis of energy eigenstates does not require that ρ be diagonal in that basis.

78. Use the time-dependent coherent state:

$$|A(t)\rangle = \sum_{n=0}^{\infty}\frac{\left(A(t)a^\dagger\right)^n}{n!}|0\rangle\, e^{-\frac{1}{2}|A(t)|^2} \quad (3)$$

to construct a density matrix $\rho(t) = |A(t)\rangle\langle A(t)|$

(a) Show that such a density matrix of this form can be found which solves Eq. (1) if we assume that $|A(t)| \gg 1$.

(b) Find the equation that $A(t)$ must obey for this to be true.

(c) Show that the resulting time-dependent expectation values of the position and momentum operators execute damped simple harmonic motion.

To prove this you will need to show that the difference of the operators $a\rho a^\dagger$ and $a^\dagger\rho a$ is a factor of $1/|A|$ smaller that either operator alone and can therefore be neglected if $|A(t)| \gg 1$. This can be shown by evaluating

$$\sum_n\langle n|\left(a\rho a^\dagger - a^\dagger\rho a\right)\left(a\rho a^\dagger - a^\dagger\rho a\right)|n\rangle \quad (4)$$

where $\rho(t) = |A(t)\rangle\langle A(t)|$.