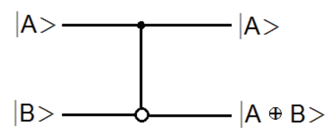
Physics GR6038 | Professor Christ
Final Examination (practice) | 5/06/2026

Answer **three (3)** of the following **four (4)** questions. Please give a complete description of your method of solution since *partial credit* will be given. (On the actual final exam there will be most likely be three problems with no choice and with each problem somewhat shorted than those below.)

1. (a) Construct the total angular momentum operator $\vec{L}$ for a system of identical, spinless bosons from the second quantized operator $\Psi(\vec{r})$ which destroys a boson at the position $\vec{r}$ and its hermitian conjugate $\Psi(\vec{r})^\dagger$ [11 points]

   (b) Find the 4×4 unitary matrix that specifies the action of the *controlled not* quantum gate on the product state $|A\rangle \otimes |B\rangle$ as described by the adjacent diagram. (Recall A and B take the values zero and one and the $\oplus$ symbol indicates an *exclusive or*.) [12 points]



   (c) What is the sign of the s-wave phase shift resulting from the low energy scattering on a weak, short-range, attractive potential? Explain. [10 points]

2. Two particles with masses M and m and positions X and y move in one dimension with the Hamiltonian

$$H = \frac{P^2}{2M} + \frac{p^2}{2m} + \frac{1}{2} m\omega_0^2 \left(1 + \frac{X^2}{R^2}\right) y^2$$

   where the length R is on the order of $\sqrt{\hbar/m\omega_0}$. Assume that $M \gg m$.

   (a) If M is treated as infinitely large and the $P^2/2M$ term removed from H, the position X becomes a classical parameter. Determine the eigenvalues and eigenvectors of H as a function of X. [5 points]

   (b) Next include the $P^2/2M$ term in H, treat X as a quantum mechanical variable and use the Born-Oppenheimer approximation to determine the ground state and ground state energy to leading order in m/M. [15 points]

   (c) Extend your results in part (b) to determine all of the eigenvectors and eigenvalues of H to leading order in m/M. [5 points]

   (d) Write down the terms in the Schrödinger equation that were neglected when determining the results found in (b) and estimate the size of the correction that would result if they were included using perturbation theory. [8 points]

3. A particle of mass M scatters in three dimensions from a weak potential V given by

$$V(\vec{r}) = \lambda e^{-\frac{\vec{r}^2}{2D^2}}.$$

   (a) Find the differential scattering cross section. [15 points]

   (b) Next assume that the particle has spin 1/2, the potential couples to its spin and is given by

$$V(\vec{r})_{m'm} = \lambda \left(\vec{\sigma}_{m'm} \cdot \vec{\nabla}\right) \left\{e^{-\frac{\vec{r}^2}{2D^2}}\right\}$$

   where $\{\sigma_i\}_{i=1,2,3}$ are the usual Pauli matrices. Find the differential cross section for the case where the initial particle is unpolarized and its polarization after scattering is not observed. [15 points]

   (c) Find the differential cross section for the case that the initial particle is polarized parallel to its momentum $\vec{p}$ and the particle's spin after scattering is also found to be parallel to the initial momentum $\vec{p}$. [10 points]

   (d) If the initial particle is polarized parallel to its momentum $\vec{p}$ and scatters through an angle θ find the direction of the scattered particle's spin. [5 points]

4. A photon of frequency ω is incident on a hydrogen atom in its ground state.

   (a) Write down the complete Hamiltonian describing the proton and electron making up the atom, the quantized E&M radiation which describes the photon and the interaction between these three components. [5 points]

   (b) Determine an expression for the scattering amplitude

$$\left\langle e^-(\vec{p}'), N(\vec{P}') \middle| T \middle| 1s(\vec{P}=0), \gamma(\vec{k}, \hat{\mathcal{E}}) \right\rangle$$

   describing the process in which the photon is absorbed and the bound electron is ejected from the atom. Here $\vec{p}\,'$ is the momentum of the outgoing electron, $\vec{P}\,'$ is the momentum of the outgoing hydrogen nucleus (here labeled as N), $\vec{P}$ the initial momentum of the atom and $\hbar\vec{k}$ the momentum of the incoming photon. (You may simplify your calculation by assuming that the proton mass is much larger than the mass of the electron, that the wave length of the photon $c/\omega \gg a_0$, where a_0 is the Bohr radius, and that the final electron velocity is larger than $e^2/\hbar$ so that the interaction of the outgoing electron and the proton can be neglected.) [15 points]

   (c) Find the differential cross section for this process, $d\sigma/d\Omega$ where Ω is the solid angle into which the electron is ejected, in a coordinate system where $\hat{z}$ is parallel to the momentum of the photon and $\hat{x}$ is parallel to the photon's polarization. [15 points]