Physics C2801 Professor Christ
due 10/01/2020

Assignment #3

19. Kleppner and Kolenkow 3.10

20. Kleppner and Kolenkow 3.17

21. Kleppner and Kolenkow 3.20

22. Kleppner and Kolenkow 4.4

23. Kleppner and Kolenkow 4.10

24. Kleppner and Kolenkow 4.13

25. Kleppner and Kolenkow 4.15 (Assume that the relative velocity u between a jumping man and the railway car is the difference between the man's and car's velocities after the jump.)

26. Find the 3×3 matrix M corresponding to a change of coordinate system from the basis $\hat{e}_1,\ \hat{e}_2,\ \hat{e}_3$ to a new basis $\hat{e}'_1,\ \hat{e}'_2,\ \hat{e}'_3$ by a rotation through θ about the $\hat{e}_2$ axis (clockwise as viewed along the direction of $\hat{e}_2$). What are the coordinates in the prime system of the vector $\vec{r} = 3\hat{e}_1 + 7\hat{e}_2 - 6\hat{e}_3$?

27. Consider a 3×3 matrix M: $M = \begin{pmatrix} M_{11} & M_{12} & M_{13} \\ M_{21} & M_{22} & M_{23} \\ M_{31} & M_{32} & M_{33} \end{pmatrix}$ and define the determinant of M by the formula:

$$\begin{aligned} \det M &= M_{11}M_{22}M_{33} + M_{12}M_{23}M_{31} + M_{13}M_{21}M_{32} \\ &- M_{13}M_{22}M_{31} - M_{12}M_{21}M_{33} - M_{11}M_{23}M_{32}. \end{aligned}$$

    (a) Show that $\det M = \det M^T$ where M^T is the transpose of M.

    (b) Using the fact that $\det(AB) = (\det A)(\det B)$ show that for an orthogonal matrix (*i.e.* $M^{-1} = M^T$), $\det M = \pm 1$.

    (c) Show that for the unit matrix: $I = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$, $\det I = +1$.

    (d) Find the matrix R that describes the change of basis vectors corresponding to the reflection: $\hat{e}'_1 = -\hat{e}_1$, $\hat{e}'_2 = -\hat{e}_2$, $\hat{e}'_3 = -\hat{e}_3$

    (e) Show that that $\det R = -1$.

    (f) Prove (perhaps using a continuity argument) that $\det M = +1$ for any matrix M describing a change of basis in which the new basis can be obtained from the original basis by performing a rotation of the original basis.