Physics UN2801 | Professor Christ
due 10/15/2020

Assignment #5

Reading:

Oct 08 Kleppner and Kolenkow Ch:6.5
Oct 13 Kleppner and Kolenkow Ch:3.7, 6.2, 6.3, 11.1, 11.2, Note 11.1
Oct 15 Kleppner and Kolenkow Ch:11.3

Problems:

36. Kleppner and Kolenkow 4.2

37. Kleppner and Kolenkow 5.7

38. Kleppner and Kolenkow 5.11

39. Kleppner and Kolenkow 5.16

40. (From Kleppner and Kolenkow 1st edition) The potential energy function for a particular two dimensional force field is given by $U(x, y) = Cxe^{-y}$, where C is a constant.

    (a) Sketch the constant energy lines.

    (b) Show that if a point is displaced by a short distance dx along a constant energy line, then its total displacement must be $\vec{dr} = dx(\hat{\imath} + \hat{\jmath}/x)$.

    (c) Using the result of b), show explicitly that $\vec{\nabla}U$ is perpendicular to the constant energy line.

41. Consider two distinct ortho-normal sets of basis vectors, $\{\hat{e}_1, \hat{e}_2, \hat{e}_3\}$ and $\{\hat{e}'_1, \hat{e}'_2, \hat{e}'_3\}$ related by a change of basis matrix, $\{M_{ij}\}_{1\le i,j\le 3}$:

$$\hat{e}'_i = \sum_{j=1}^{3} M_{ij}\hat{e}_j.$$

Using these two coordinate systems a vector $\vec{A}$ can be written in terms of two sets of coordinate: $\{a_i\}_{1\le i\le 3}$ and $\{a'_i\}_{1\le i\le 3}$:

$$\vec{A} \;=\; \sum_{i=1}^{3} a_i\,\hat{e}_i \quad \text{and} \quad \vec{A} = \sum_{i=1}^{3} a'_i\,\hat{e}'_i$$

where

$$a'_i = \sum_{j=1}^{3} M_{ij}a_j. \qquad (1)$$

Using the chain rule relate the two sets of partial derivatives: $\{\partial V(\vec{r})/\partial r_i\}_{1\le i\le 3}$ and $\{\partial V(\vec{r}(\vec{r}\,'))/\partial r'_i\}_{1\le i\le 3}$ showing that these three quantities transform between systems like the coordinates of a vector, as in Eq. (1). (Recall that $M^{-1} = M^t$.)

42. Kleppner and Kolenkow 6.5

43. Kleppner and Kolenkow 6.8