9/8/20 (1)

Accelerated Physics UN2801/2

About the course:

- Most information on an open website
  www.columbia.edu/~nhc/UN2801
  $\underbrace{\text{~nhc/UN2801}}_{\text{case matters}}$
- Recitation sessions:
  5:10 PM Wed. } move one
  7:10 PM Thur. } to morning!
- Office hours
  9:15 AM Mon. } problem solving?
  3:10 PM Fri } special topics?
  becoming a physicist?
  groups with special interests?
  your ideas?
- Textbook: Kleppner & Kolenkow
  Introduction to Mechanics
  2nd edition
  Other books on the web site
- Problems should be uploaded to Courseworks on Thursday
- Grade: 20% homework, 30% midterm, 50% final

I Newtonian dynamics

A 1-dim motion

1. Kinematics - describe motion

- consider a block moving in one dimension
- locate the block by giving its distance to a fixed point P

- x is a real number
- $|x|$ is the distance between left edge of the block and P
- $x>0$, block is to the right of P
- $x<0$, " " " " left of P
- Actually x cannot be a pure number since a measure of distance must have a unit of distance, e.g. a "meter".
- Thus x must be a product of a number times a unit:
  $x = 8.6$ meters.

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  $x = 8.6$ meters.

% final

- Convention that distance is the product of a number times a unit works very well:

We can easily change units:

1 meter = 100 cm

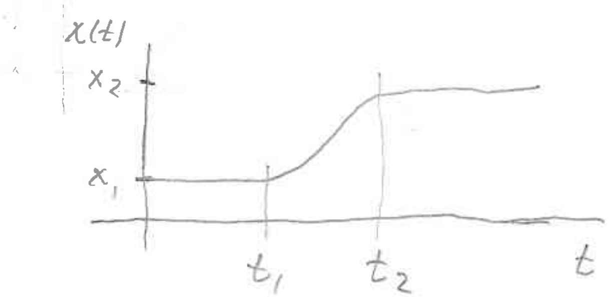
(express distance of 1 meter in units of cm) then

$$x = 8.6\text{ m} = 8.6\times(100\text{ cm}) = 860\text{ cm}$$

$$\text{or } x = 8.6\text{ m}\times\underbrace{\frac{100\text{ cm}}{1\text{ m}}}_{1} = 860\text{ cm}$$

! note we MUST ALWAYS give the unit !

- To describe motion we need only say how x depends on time t (another product of a real number and a unit): $x(t)$



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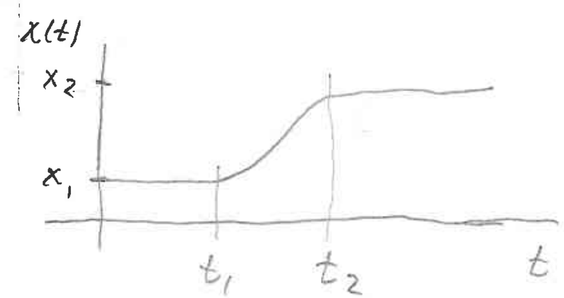
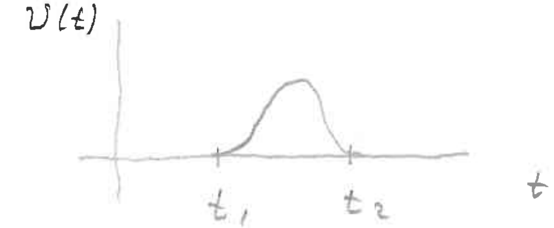


the velocity $v(t) = \frac{dx}{dt}(t) = \lim_{\Delta t \to 0} \frac{x(t+\Delta t) - x(t)}{\Delta t}$

is the rate of change of position w.r.t. time at the time t, the slope of the curve $x(t)$ at t:



finally the acceleration $a(t) = \frac{dv}{dt}(t)$

2. Dynamics - Predict motion
   - introduce the concept of a force: a push or pull acting on our block
   - Newton's 1st law: If no forces act on our block $v(t) = $ constant, $a(t) = 0$.

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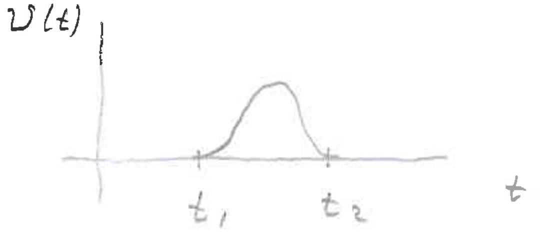

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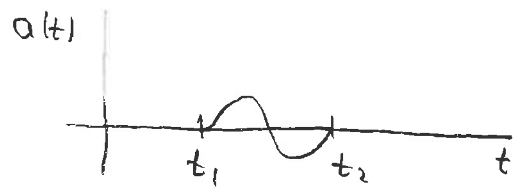

2. Dynamics - predict motion

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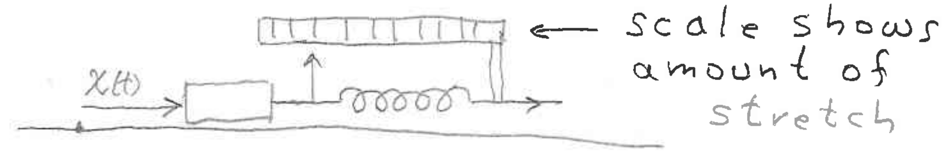
- Next apply a constant force to the block. For example attach a spring stretched by a fixed amount



Discover:

- $a(t)$ is a non-zero constant
- if we enlarge the block by adding additional material of the same type, increasing its volume by a factor r, the new $a_r(t) = \frac{1}{r} a(t)$

- We can use our "constant force" F to define what is known as the <u>inertial mass</u> of any block:
  - Agree on a standard block defined to have mass 1 kg.
  - Apply F to 1 kg and to block B define $m_B = \frac{a_{1\,kg}}{a_B} \times 1\ kg$

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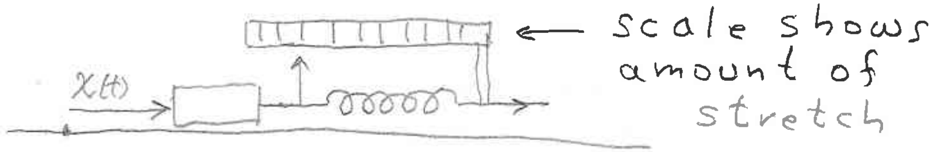

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  - Agree on a standard block defined to have mass 1 kg.
  - Apply F to 1 kg and to block B

define $m_B = \dfrac{a_{1kg}}{a_B} \times 1\text{ kg}$

We have discussed how the acceleration produced by our force depends on the block and used it to determine the mass of any block in kilograms (kg). How does the acceleration depend on the force?

- Our definition $m_B = \dfrac{a_{1kg}}{a_B} 1\text{ kg}$

may depend on our choice of force! Experiment shows it does not. All forces will define the same inertial masses. We can define

$$F = m_B a_B = a_{1KG} \cdot 1\text{ kg}$$

using any block B. Each force is characterized by a number F with units $\text{kGm}/(\text{sec})^2 \equiv \text{Newton}$,

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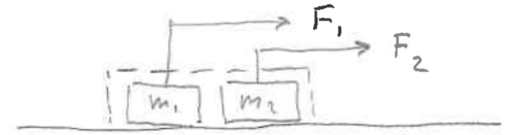
How do forces combine?

Divide M into two parts $M = m_1 + m_2$ with $m_1/F_1 = m_2/F_2$. Try a similar setup

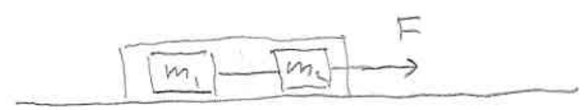


$a = F_1/m_1 = F_2/m_2$. Must also be a for our original set up. Thus

$$Ma = (m_1 + m_2)a = m_1 a + m_2 a = F_1 + F_2$$

and we should expect forces to add!

Deduce Newton's third law:



$\rightarrow F_{2\,on\,1}$

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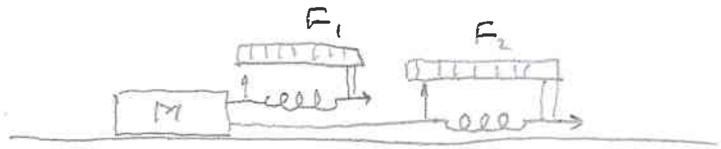

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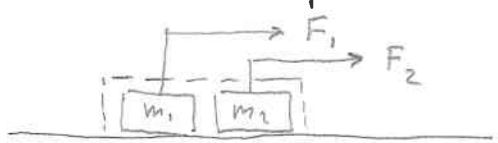

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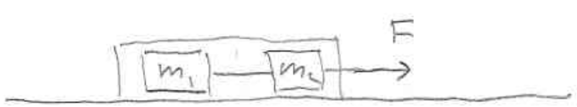

$\rightarrow F_{2\,on\,1}$

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The combined system must accelerate with $a = \dfrac{F}{m_1+m_2}$

Thus $F_{2\,on\,1} = m_1 a$

$$F_{1\,on\,2} + F = m_2 a$$

$$F_{1\,on\,2} + (m_1+m_2)\,a = m_2 a$$

or $F_{1\,on\,2} = -m_1 a = -F_{2\,on\,1}$

Newton's third law: If body 1 exerts a force F on body 2, the 2 exerts a force $-F$ on 1.

Note: The fact that masses and forces add so simply should not be a surprise. These are the visible results of the smaller atoms whose masses and forces add to produce the classical macroscopic physics we are studying.

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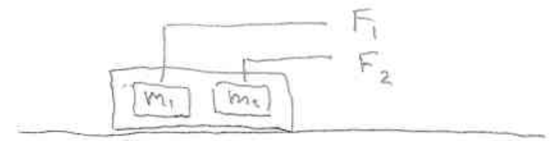
Addendum



$$F_1 + F_2 = (m_1 + m_2)\, a$$

- We showed the effect of combining forces was additive since we stated experiment showed $m = m_1 + m_2$ when combining blocks of the same material

- If m_1 & m_2 are made of different materials, we can reverse the above argument. Choose F_1 & F_2 with $F_1/m_1 = F_2/m_2 = a$ and use diagram above to argue that

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a will be acceleration of m_1 & m_2 together so that $m_1 + m_2$ must be its mass.

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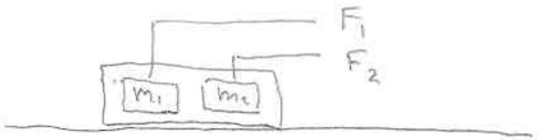

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3 Examples

a) A block of mass m falls starting a rest from a height h. If it hits the ground in 3 sec, how high was it.

Need to know the force of gravity at the earth's surface $F = mg$ with $g = 9.8 \frac{m}{sec^2}$. Thus $a = -g$

$$\frac{dv}{dt} = -g, \quad v(0) = 0 \Rightarrow v(t) = -gt$$

$$\frac{d}{dt} x(t) = -gt \text{ and } x(0) = h \Rightarrow x(t) = h - \tfrac{1}{2} g t^2$$

$$\& \; x(3\,sec) = 0 = h - \tfrac{1}{2} g\, (3\,sec)^2$$

$$\text{or } h = \tfrac{1}{2} \times 9.8 \frac{m}{sec^2} \times 9\, sec^2 = 44.1\, m$$

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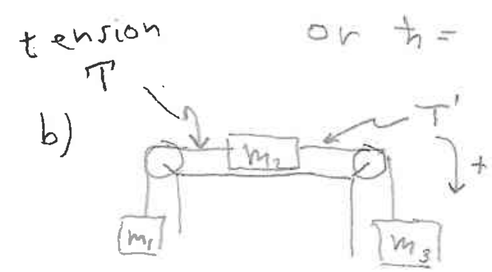

$$\left.\begin{aligned} m_1 a &= T - m_1 g \\ m_2 a &= T' - T \\ m_3 a &= m_3 g - T' \end{aligned}\right\} \text{add}$$

find a

$$(m_1 + m_2 + m_3)\, a = (m_3 - m_1)\, g$$

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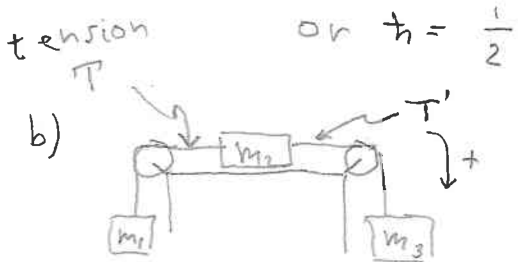

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4. More about gravity

$F = m_I a$ defines the inertial mass

The force of gravity near the earth's surface might define a different mass: $F_{grav} = m_{grav} \cdot g$

where we might determine g by requiring $m_{grav} = m_I$ for gold.

Eötvös experiment showed $m_{grav} = m_I$ to one part per 10^8.

Explained by Einstein's equivalence principle - the basis of general relativity.

5. Time varying forces

expect at a time t $F(t) = m\, a(t)$

since for a very short time scale $F(t)$ will appear to be constant

must solve $\frac{d^2x}{dt^2} = \frac{F(t)}{m}$

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Can we solve for $x(t)$ if we know

$$\frac{d^2x}{dt^2} = \frac{F(t)}{m} \quad ?$$

Yes! If we are given $x(0)$ and $\dfrac{dx}{dt}(0)$

Write Newton's law as two coupled 1st order equations

$$\frac{dx}{dt} = v(t) \quad \& \quad \frac{dv}{dt} = \frac{F(t)}{m}$$

Divide time into N small intervals of length Δt and use

$$x(\Delta t) = x(0) + v(0)\,\Delta t + O(\Delta t^2)$$
$$v(\Delta t) = v(0) + \frac{F(0)}{m}\,\Delta t + O(\Delta t^2)$$

Each is an application of a Taylor series:

$$f(t+\Delta t) = f(t) + \frac{df}{dt}(t)\,\Delta t + \frac{1}{2}\frac{d^2f}{dt^2}(\Delta t)^3 + \frac{1}{3!}\frac{d^3f}{dt^3}(\Delta t)^3 + \dots$$

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Can repeat to find $x(n\Delta t)$ & $v(n\Delta t)$

$$x(n\Delta t) = x((n-1)\Delta t) + v((n-1)\Delta t)\,\Delta t$$

$$v(n\Delta t) = v((n-1)\Delta t) + \frac{F((n-1)\Delta t)}{m}\,\Delta t$$

Error in $x(n\Delta t)$ or $v(n\Delta t)$ will be $(\Delta t)^2$ error at each step $\times$ n steps

error $\sim (\Delta t)^2 \times n = (n\Delta t)\,\Delta t = t \cdot \Delta t$