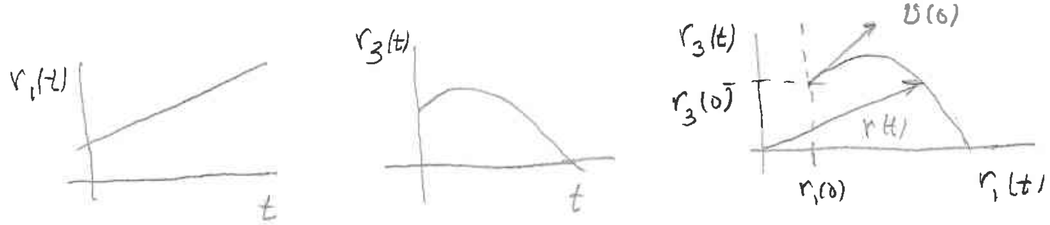
$$r_1(t) = r_1(0) + v_1(0)\,t$$

$$r_2(t) = r_2(0)$$

$$r_3(t) = r_3(0) + v_3(0)\,t - \tfrac{1}{2} g t^2$$

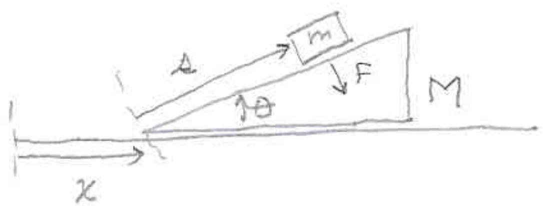


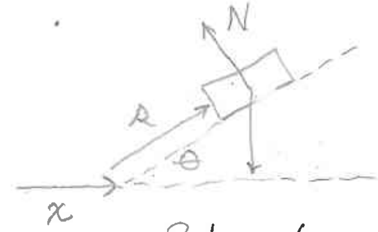
$$\vec{v}(0) = v_1(0)\,\hat{e}_1 + v_3(0)\,\hat{e}_3$$

tangent to $\vec{r}(t)$ at $t=0$

$$\left.\frac{\Delta r_3(t)}{\Delta r_1(t)}\right|_{t=0} = \frac{v_3(0)\,\Delta t}{v_1(0)\,\Delta t} = \frac{v_3(0)}{v_1(0)}$$

b) A block of mass m slides on a wedge of mass M which slides on a horizontal table. If both move without friction find the acceleration of the wedge





Block

Wedge

Veritical forces and acceleration:

$$m\ddot{s}\sin\theta = N\cos\theta - mg$$

Horizontal

$$-m(\ddot{x} + \ddot{s}\cos\theta) = -N\sin\theta$$

$\times \sin\theta$

$\times(-\cos\theta)$

$$m\sin\theta\,\ddot{x} = -N + mg\cos\theta$$

$$M\ddot{x}\,\frac{1}{\sin\theta}$$

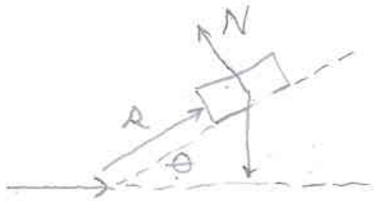
Horizontal forces and acceleration:

$$M\ddot{x} = N\sin\theta$$

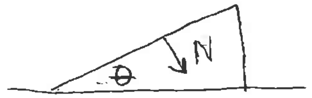
3 equations,
3 unknowns

$$(m\sin^2\theta + M)\,\ddot{x} = mg\sin\theta\cos\theta$$

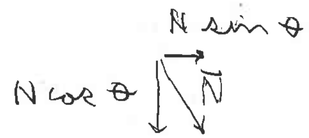
$$\ddot{x} = \frac{m\sin\theta\cos\theta}{m\sin^2\theta + M}\,g$$



Block



Wedge



Veritical forces and acceleration:

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Horizontal forces and acceleration:

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3 equations, 3 unknowns

$\times\sin\theta$

$\times(-\cos\theta)$

$$m\sin\theta\,\ddot{x} = -N + mg\cos\theta$$

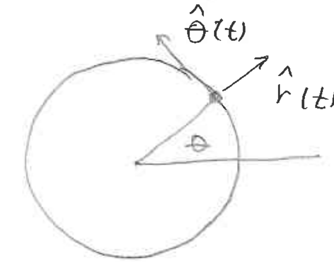
$$M\ddot{x}\frac{1}{\sin\theta}$$

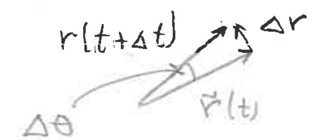
$$(m\sin^2\theta + M)\ddot{x} = mg\sin\theta\cos\theta$$

$$\ddot{x} = \frac{m\sin\theta\cos\theta}{m\sin^2\theta + M}g$$

6. Circular motion – two ways

a) abstract vectors



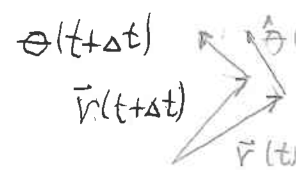


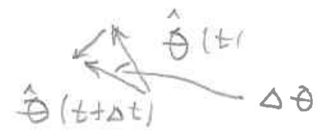
$$\Delta\vec{r} = \hat\theta\cdot r\cdot\underbrace{\Delta\theta}_{\dot\theta\,\Delta t}$$

$$\vec{v}(t) = \frac{\Delta\vec r}{\Delta t} = \hat\theta\, r\dot\theta = \hat\theta\, r\,\omega(t) \quad \text{if } \dot\theta = \omega$$

$$\vec{a}(t) = \frac{d\vec v}{dt} = \frac{d}{dt}\{\hat\theta(t)\, r\,\omega(t)\}$$

$$= \frac{d\hat\theta}{dt} r\,\omega(t) + \hat\theta\, r\,\dot\omega(t)$$





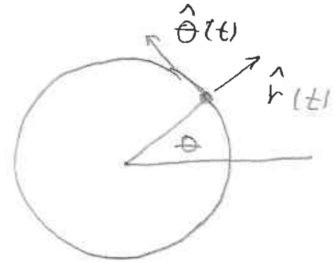
$$\frac{d\hat\theta}{dt} = -\hat r\frac{\Delta\theta}{\Delta t} = -\hat r\,\omega$$

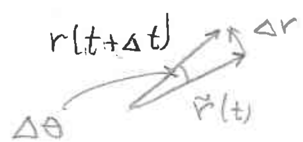
$$\Rightarrow \vec a = \underbrace{-\vec r\,\omega^2}_{\text{centripetal acceleration}} + r\hat\theta\,\dot\omega$$

Particle traveling in a circle must be pulled inward by a centripetal force $\vec F = -\vec r\,\omega^2 m$

6. Circular motion – two ways

a) abstract vectors



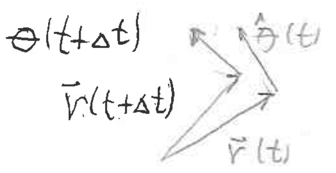


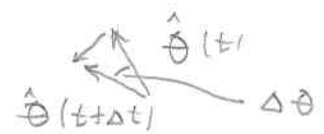
$$\Delta \vec{r} = \hat{\theta} \cdot r \cdot \underbrace{\Delta\theta}_{\dot{\theta}\,\Delta t}$$

$$\vec{v}(t) = \frac{\Delta \vec{r}}{\Delta t} = \hat{\theta}\, r\, \dot{\theta} = \hat{\theta}\, r\, \omega(t) \quad \text{if } \dot{\theta} = \omega$$

$$\vec{a}(t) = \frac{d\vec{v}}{dt} = \frac{d}{dt}\left\{\hat{\theta}(t)\, r\, \omega(t)\right\}$$

$$= \frac{d\hat{\theta}}{dt}\, r\, \omega(t) + \hat{\theta}\, r\, \dot{\omega}(t)$$



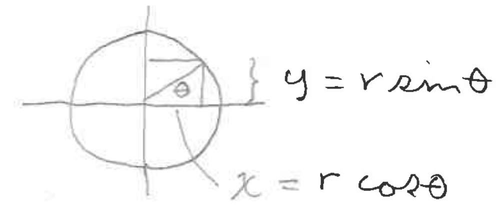


$$\frac{d\hat{\theta}}{dt} = -\hat{r}\,\frac{\Delta\theta}{\Delta t} = -\hat{r}\,\omega$$

$$\text{\&}\quad \vec{a} = \underbrace{-\vec{r}\,\omega^2}_{\text{centripetal acceleration}} + r\,\hat{\theta}\,\dot{\omega}$$

Particle traveling in a circle must be pulled inward by a centripetal force $\vec{F} = -\vec{r}\,\omega^2 m$

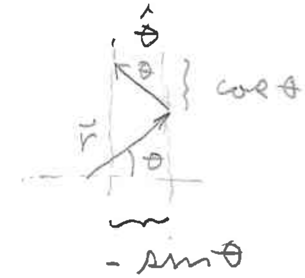
b) coordinates



$$\vec{v} = \frac{d}{dt}(r\cos\theta,\ r\sin\theta)$$

$$= \left(+r\sin\theta\,\frac{d\theta}{dt},\ r\cos\theta\,\frac{d\theta}{dt}\right)$$

$$= r\,\frac{d\theta}{dt}\,\underbrace{(-\sin\theta,\ \cos\theta)}_{\hat{\theta}}$$

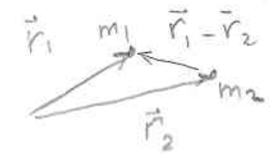


$$\vec{a} = \frac{dv}{dt} = \frac{d}{dt}\, r\,\omega(t)\,(-\sin\theta(t),\ \cos\theta(t))$$

$$= r\,\dot{\omega}\,\hat{\theta} + r\,\omega^2(t)\,(-\cos\theta(t),\ -\sin\theta(t))$$

$$= r\,\dot{\omega}\,\hat{\theta} - \vec{r}\,\omega^2 \ \checkmark$$

7. Newton's law of gravitation



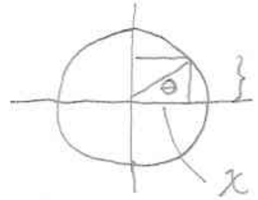
$$\vec{F}_{1 \text{ on } 2} = \frac{G m_1 m_2}{|r_1 - r_2|^2}\,\frac{\vec{r}_1 - \vec{r}_2}{|\vec{r}_1 - \vec{r}_2|}$$

$$G = 6.67\times 10^{-8}\ \frac{\text{dyne}\cdot\text{cm}^2}{\text{gm}}$$

recall $1 \text{ Newton} = \dfrac{\text{kg m}}{\text{sec}^2}$

$$1 \text{ dyne} = \frac{\text{g}\cdot\text{cm}}{\text{sec}^2} = 10^{-5}\ \text{Newton}$$

b) coordinates

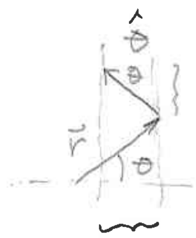


$y = r\sin\theta$, $x = r\cos\theta$

$$\vec{v} = \frac{d}{dt}(r\cos\theta, r\sin\theta)$$

$$= \left(-r\sin\theta \frac{d\theta}{dt}, r\cos\theta\frac{d\theta}{dt}\right)$$

$$= r\frac{d\theta}{dt}\underbrace{(-\sin\theta, \cos\theta)}_{\hat{\theta}}$$



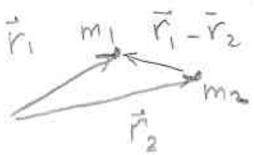
$\cos\theta$, $-\sin\theta$

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt} r\omega(t)(-\sin\theta(t), \cos\theta(t))$$

$$= r\dot{\omega}\hat{\theta} + r\omega^2(t)(-\cos\theta(t), -\sin\theta(t))$$

$$= r\dot{\omega}\hat{\theta} - \hat{r}\omega^2 \checkmark$$

7. Newton's law of gravitation



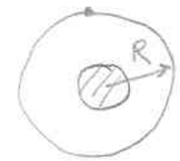
$$\vec{F}_{1\text{ on }2} = \frac{Gm_1m_2}{|r_1-r_2|^2}\frac{\vec{r}_1-\vec{r}_2}{|\vec{r}_1-\vec{r}_2|}$$

$$G = 6.67\times10^{-8}\ \frac{\text{dyne}\cdot\text{cm}^2}{\text{gm}}$$

recall $1\ \text{Newton} = \frac{\text{kg m}}{\text{sec}^2}$

$$1\ \text{dyne} = \frac{\text{g}\cdot\text{cm}}{\text{sec}^2} = 10^{-5}\ \text{Newton}$$

Problem: Find the radius of a geosynchrous orbit



$$\omega = \frac{2\pi}{\text{day}}$$

$$\cancel{m_1}R\omega^2 = \frac{G\cancel{m_1}M_E}{R^2}$$

$$R^3 = \frac{GM_E}{\omega^2} = \underbrace{\frac{GM_E}{R_E^2}}_{g}\times\frac{R_E^2}{\omega^2}$$

$$= \frac{32\ \text{ft/sec}^2 \times (3960\ \text{mi})^2}{(2\pi)^2}(24\times3600\ \text{sec})^2 \times \frac{1\ \text{mi}}{5280\ \text{ft}}$$

$$R = \left[\frac{32\times(3960)^2(24\times3600)^2}{(2\pi)^2\ 5280}\right]^{1/3}\text{mi}$$

$$= 26{,}200\ \text{mi}$$

8. Changing basis vectors

$$\vec{A} = a_1\hat{e}_1 + a_2\hat{e}_2 + a_3\hat{e}_3$$

$$= a_1'\hat{e}_1' + a_2'\hat{e}_2' + a_3'\hat{e}_3'$$

- Change to a more convenient basis
- Check that physical law does not depend on choice of basis

Problem: Find the radius of a geosynchrous orbit

$$\omega = \frac{2\pi}{\text{day}}$$

$$\cancel{m_1} R \omega^2 = \frac{G \cancel{m_1} M_E}{R^2}$$

$$R^3 = \frac{G M_E}{\omega^2} = \underbrace{\frac{G M_E}{R_E^2}}_{g} \times \frac{R_E^2}{\omega^2}$$

$$= \frac{32\ \text{ft/sec}^2 \times (3960\ \text{mi})^2}{(2\pi)^2} (24 \times 3600\ \text{sec})^2 \times \frac{1\ \text{mi}}{5280\ \text{ft}}$$

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8. Changing basis vectors

$$\vec{A} = a_1 \hat{e}_1 + a_2 \hat{e}_2 + a_3 \hat{e}_3$$

$$= a_1' \hat{e}_1' + a_2' \hat{e}_2' + a_3' \hat{e}_3'$$

- Change to a more convenient basis
- Check that physical law does not depend on choice of basis

Express the new basis in terms of the old one

$$\hat{e}_i' = M_{i1} \hat{e}_1 + M_{i2} \hat{e}_2 + M_{i3} \hat{e}_3 \qquad i = 1, 2 \;\&\; 3$$

$$\text{note} \quad \hat{e}_i' \cdot \hat{e}_j = M_{ij}$$

Then $a_i' = \hat{e}_i' \cdot A = \hat{e}_i' \cdot [a_1 \hat{e}_1 + a_2 \hat{e}_2 + a_3 \hat{e}_3]$

$$= M_{i1} a_1 + M_{i2} a_2 + M_{i3} a_3$$

Thus the a_i' are given by matrix multiplication

$$\begin{pmatrix} a_1' \\ a_2' \\ a_3' \end{pmatrix} = \begin{pmatrix} M_{11} & M_{12} & M_{13} \\ M_{21} & M_{22} & M_{23} \\ M_{31} & M_{32} & M_{33} \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$$

Recall in general

$$\underset{l \times m}{X} \cdot \underset{m \times k}{Y} = \underset{l \times k}{(XY)}$$

$$(XY)_{ab} = \sum_{c=1}^{m} X_{ac} Y_{cb} \qquad \begin{array}{l} 1 \le a \le l \\ 1 \le b \le k \end{array}$$

Next change basis twice

$$A = \sum_{i=1}^{3} a_i \hat{e}_i = \sum_{i=1}^{3} a_i' \hat{e}_i' = \sum_{i=1}^{3} a_i'' \hat{e}_i''$$