Location of interference friges is determined by

$$t_1 - t_2 = \frac{2l_1}{c}\frac{1}{1-v^2/c^2} - \frac{2l_2}{c}\frac{1}{\sqrt{1-v^2/c^2}}$$

Rotate apparatus 90° clock wise

$$(t_1 - t_2)_{90°} = \frac{2l_1}{c}\frac{1}{\sqrt{1-v^2/c^2}} - \frac{2l_2}{c}\frac{1}{1-v^2/c^2}$$

This rotation will shift the difference between there times by

$$(t_1 - t_2) - (t_1 - t_2)_{90°} = \frac{2(l_1+l_2)}{c}\left[\frac{1}{1-v^2/c^2} - \frac{1}{\sqrt{1-v^2/c^2}}\right]$$

$$\approx \frac{2(l_1+l_2)}{c}\left[1 + v^2/c^2 - \left(1 + \frac{v^2}{2c^2}\right)\right] = \frac{l_1+l_2}{c} \times \frac{v^2}{c^2}$$

$$\sim \frac{10\ m}{3\times10^8\ m/sec} \times (10^{-4})^2 = 3\times10^{-15}\ sec$$

v/c from earlier

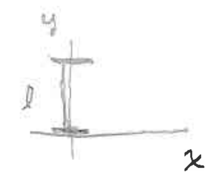
One interference fringe $\equiv$ one period of light oscilation $= \frac{\lambda}{c} = \frac{10^{-6} m}{3\times10^{-8}\ m/sec} \sim 3\times10^{-15}$ sec

should see pattern move by one fringe.

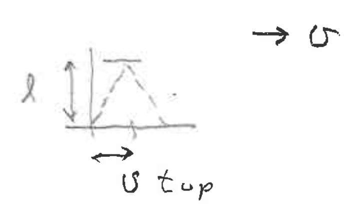
It does not move!

B Adopt Einstein's "principle of relativity": light moves at the speed c in all inertial reference frames. What are the implications of this assumption?

1. Time Design a simple clock using light propagation to measure time:



use time for light to travel up and back as standard of time $\tau = 2l/c$. Next look at a clock moving to the right in the x direction with speed v



$$t_{up}\, c = \sqrt{l^2 + (v\, t_{up})^2}$$

$$t_{up}^2 (c^2 - v^2) = l^2$$

$$\text{period} = \frac{2l}{c}\frac{1}{\sqrt{1-v^2/c^2}}$$

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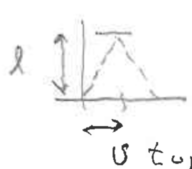

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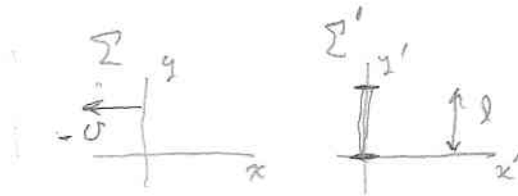
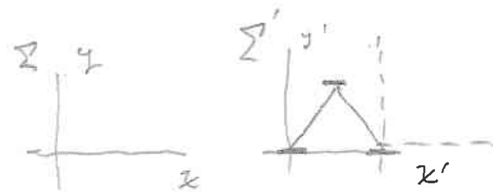
Moving and stationary observers watch the same moving clock but deduce that different times have elapsed.

Moving observer sees $\Delta t' = \frac{2l}{c}$



Stationary observer see $\Delta t = \frac{2l}{c} \frac{1}{\sqrt{1-v^2/c^2}}$



$t' = 0 \qquad t' = \frac{2l}{c}$

Σ must see all phenomena that measure time that are stationary in Σ' slowed by the same factor $\gamma = \frac{1}{\sqrt{1-v^2/c^2}}$

Note Σ & Σ' must see the same vertical length l, perpendicular to their parallel x & x' axes.

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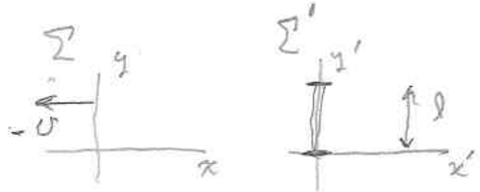

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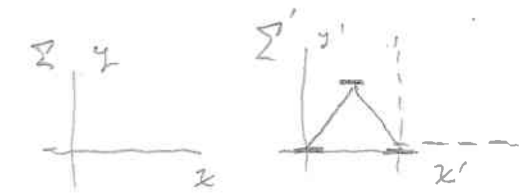

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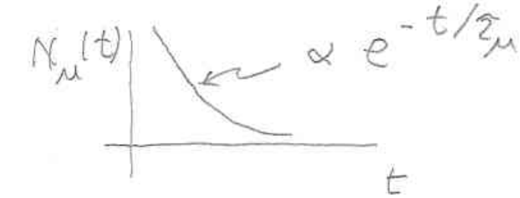
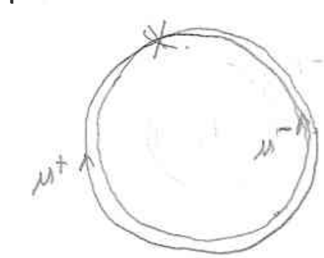
Consider a muon $\mu^{\pm}$ an elementary particle with mass $m_\mu = 105$ MeV about 200 x heavier than an electron which is unstable: $\mu^- \to e^- + \bar{\nu}_e + \nu_\mu$ with a lifetime $\tau_\mu = 2 \times 10^{-6}$ sec for muons at rest. What is their lifetime if $v_\mu = c/2$?



$$\tau_{lab} = \gamma \tau_\mu = \frac{1}{\sqrt{1-v^2/c^2}}\tau_\mu = \frac{2\times10^{-6}}{\sqrt{1-\left(\frac{1}{2}\right)^2}} = 2\times10^{-6}\sqrt{\frac{4}{3}}$$

$$= \frac{4}{\sqrt{3}} \times 10^{-6}\text{ sec} = 2.3 \times 10^{-6}\text{ sec}$$

A storage ring with $\gamma = 1000$ would allow experiments with coliding muon beams



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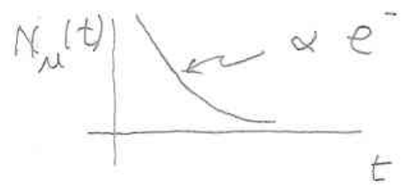

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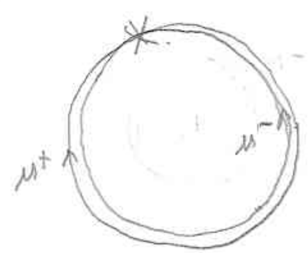

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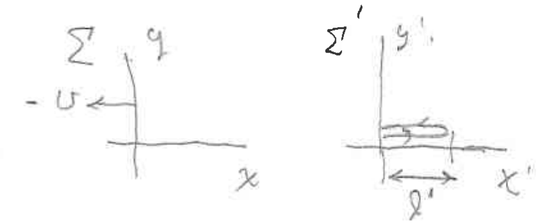
2. Moving lengths parallel to v

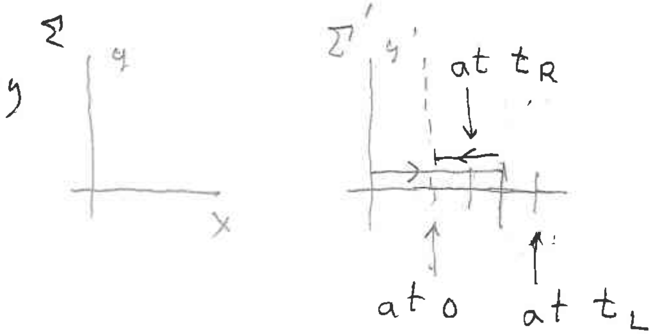
Moving observer



Σ' say

$\Delta t' = \frac{2\bar{l}'}{c}$

Stationary observer



$c t_R = \bar{l} + v t_R \qquad t_R = \frac{\bar{l}}{c-v}$

$c t_L = \bar{l} - v t_L \qquad t_L = \frac{\bar{l}}{c+v}$

$$\Delta t = t_R + t_L = \frac{2c\bar{l}}{c^2-v^2} = \frac{2\bar{l}}{c}\,\frac{1}{1-v^2/c^2}$$

but $\Delta t = \frac{\Delta t'}{\sqrt{1-v^2/c^2}}$

or $\frac{2\bar{l}}{c}\,\frac{1}{1-v^2/c^2} = \frac{2\bar{l}'}{c}\,\frac{1}{\sqrt{1-v^2/c^2}}$

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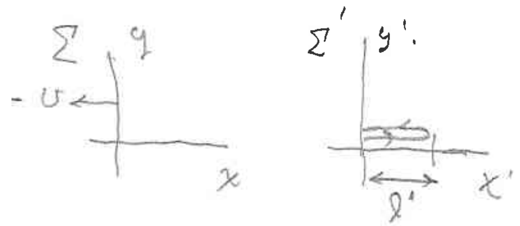
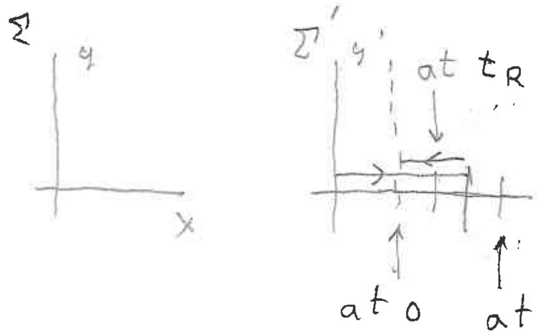


Σ' say $\bar{l}' - \bar{l}$

$$\Delta t' = \frac{2\bar{l}'}{c}$$

Stationary observer



$$c t_R = \bar{l} + v t_R \qquad t_R = \frac{\bar{l}}{c-v}$$

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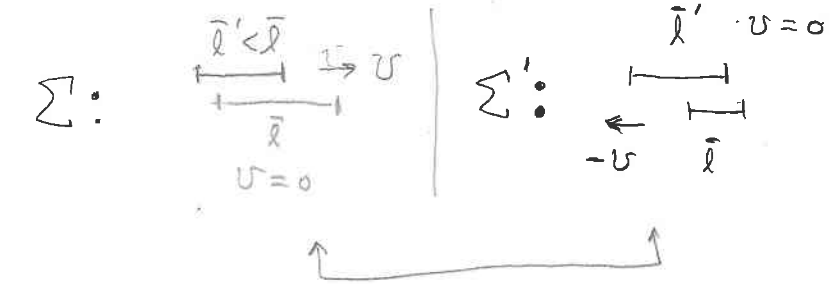
3. Why can't we compare $\bar{l}$ & $\bar{l}'$ as the pass over each other

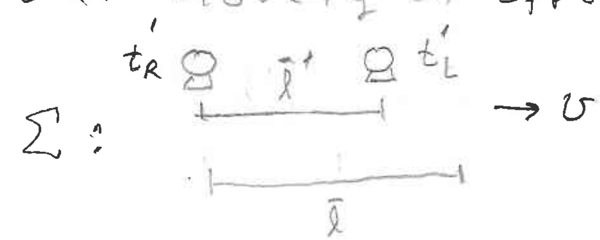


These two comparisons appear in conflict !?

Σ says left ends pass each other before the right ends

Σ' says the right ends pass each other before the left ends

This is OK since each sees the other moving in opposite direction!



$t'_L < t'_R$

Synchronous clocks in Σ' not so in Σ

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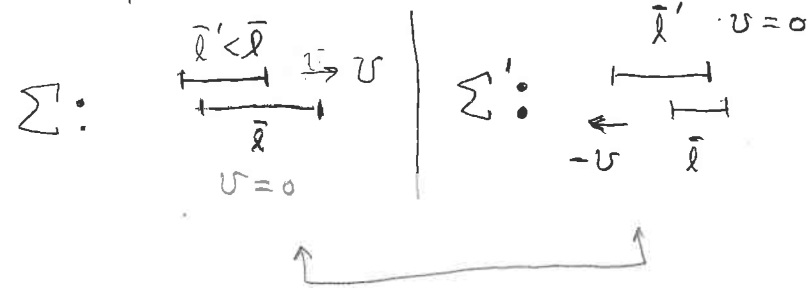

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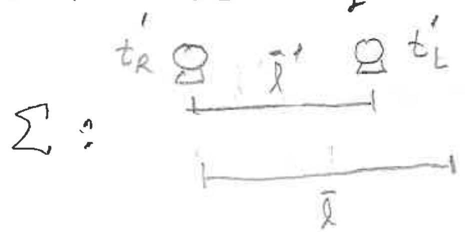
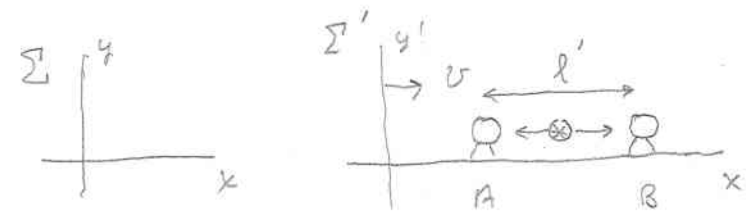


$\rightarrow v \qquad t_L' < t_R'$

Synchronous clocks in Σ' not so in Σ

Easier to work out from a different setup:



When illuminate by a flash from a light midway between, both A & B read the same time

$\Rightarrow$ synchronous is Σ'

Σ says if flash occurs at $t = 0$ then A & B are illuminated at $t_A \neq t_B$

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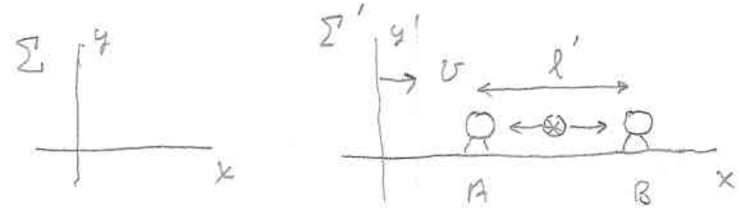

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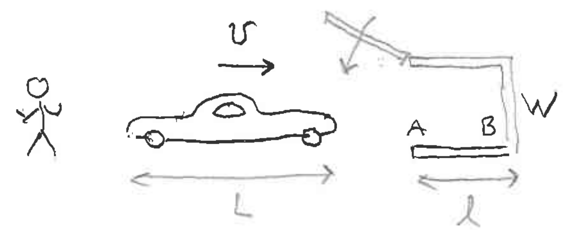
Clock which are synchronous in a moving frame are not synchronous when viewed by a statoonary observer. The down stream clock reads the same time as the up stream clock $\frac{l}{c}\gamma$ later.

Solves parking garage paradox



$L > l$

But parking fee collect when reckless parking attendent drives car so fast that $L/\gamma < l$ and quickly closes garage door.

- Stationary car owner see stationary clocks at A+B read the same time when door closes but car has not hit wall W.
- Driver sees collision with the wall before the door closes. A reads an earlier time than B