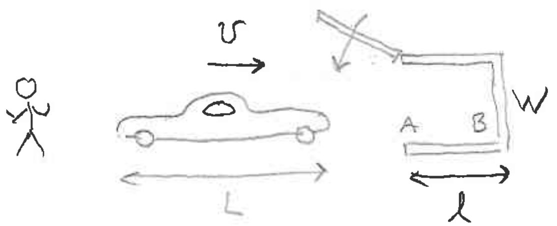
Clock which are synchronous in a moving frame are not synchronous when viewed by a statoonary observer. The down stream clock reads the same time as the up stream clock $\frac{l}{c}\gamma$ later.

Solves parking garage paradox



$L > l$

But parking fee collect when reckless parking attendent drives car so fast that $L/\gamma < l$ and quickly closes garage door.

- Stationary car owner see stationary clocks at A+B read the same time when door closes but car has not hit wall W.
- Driver sees collision with the wall before the door closes. A reads an earlier time than B

Reasoning about slow, asynchronous clocks and shrinking lengths can get very confusing.

Describe as a change of coorinates

4. Lorentz transformations

$$\tau = \tau'\gamma \qquad l = l'/\gamma \qquad + \quad \Delta t = \frac{v}{c^2} l'\gamma$$

contains everything. Express it more precisely - USE MORE MATH

For an observer in a reference system Σ introduce a space-time coordinate system to locate an "event", something that happens at a specific point and time

① Set up a standard 3-D coordinate system (x, y, z) and vectors $\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$ to locate any point in space.

Reasoning about slow, asynchronous clocks and shrinking lengths can get very confusing.

Describe as a change of coorinates

4. Lorentz transformations

$$\tau = \tau'\gamma \qquad l = l'/\gamma \qquad + \quad \Delta t = \frac{v}{c^2}\, l'\gamma$$

contains everything. Express it more precisely - USE MORE MATH

For an observer in a reference system Σ introduce a space-time coordinate system to locate an "event", something that happens at a specific point and time

① Set up a standard 3-D coordinate system (x, y, z) and vectors $\vec{r} = x\,\hat{\imath} + y\,\hat{\jmath} + z\,\hat{k}$ to locate any point in space.

② Fill space with a 3-D array of clocks so there is a clock very close to every point in space. These clocks all run at the same rate and are synchronous in Σ

- An event is located by (x, y, z, t) where t is the time shown on the clock at (x, y, z) when the event occurs.
- All observed will agree that the clock reads t since these two events happen at the same place.

Next consider a moving observer who sets up a similar coordinate system Σ' and locates the event by (x', y', z', t')

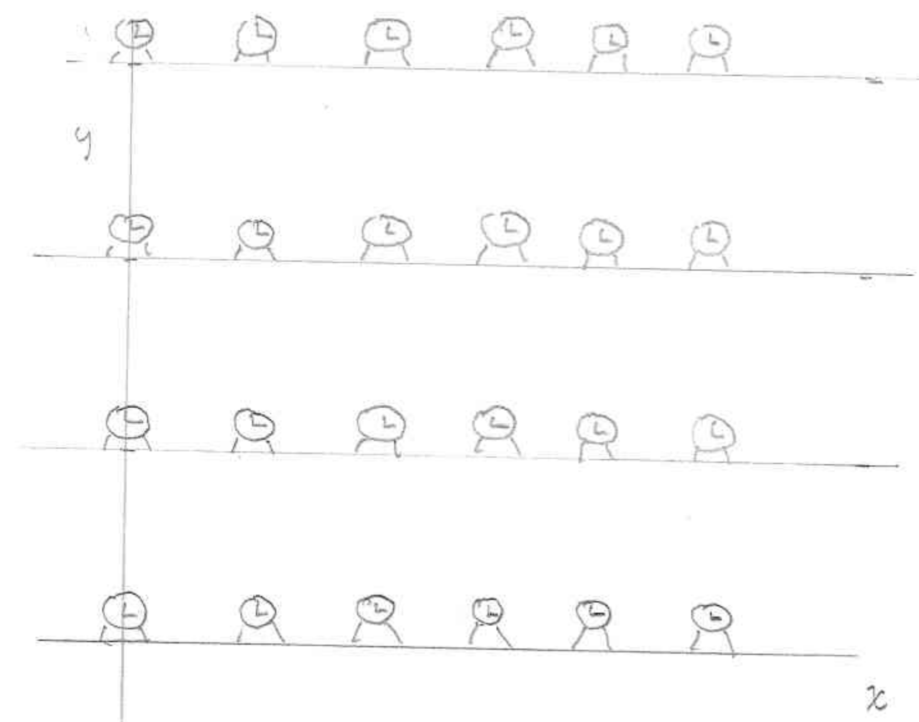
(2) Fill space with a 3-D array of clocks so there is a clock very close to every point in space. These clocks all run at the same rate and are synchronous in Σ

- An event is located by (x, y, z, t) where t is the time shown on the clock at (x, y, z) when the event occurs.
- All observed will agree that the clock reads t since these two events happen at the same place.

Next consider a moving observer who sets up a similar coordinate system Σ' and locates the event by (x', y', z', t')

← Add a 3-D array of clocks to usual 3-D coordinates

Σ



② Fill space with a 3-D array of clocks so there is a clock very close to every point in space. These clocks all run at the same rate and are synchronous in Σ

- An event is located by (x, y, z, t) where t is the time shown on the clock at (x, y, z) when the event occurs.
- All observed will agree that the clock reads t since these two events happen at the same place.

Next consider a moving observer who sets up a similar coordinate system Σ' and locates the event by (x', y', z', t')

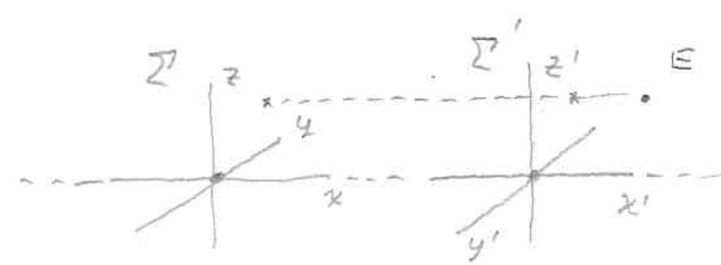
Assume that these two system are related as follows

a) The Σ' system move with speed v as seen by Σ in the $+\hat{\imath}$ direction

b) The Σ system moves with speed v as seen by Σ' in the $-\hat{\imath}'$ direction

c) The origins $(0,0,0)$ in Σ & Σ' pass through each other an coincide when $t = t' = 0$

d) The y and y' axes and z and z' axes pass over each other and are parallel



Assume that these two systems are related as follows

a) The Σ' system move with speed v as seen by Σ in the $+\hat{\imath}$ direction

b) The Σ system moves with speed v as seen by Σ' in the $-\hat{\imath}'$ direction

c) The origins $(0,0,0)$ in Σ & Σ' pass through each other an coincide when $t = t' = 0$

d) The y and y' axes and z and z' axes pass over each other and are parallel

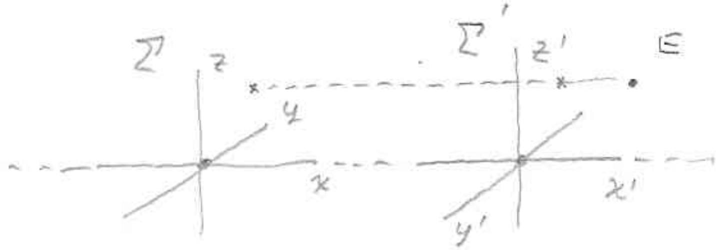
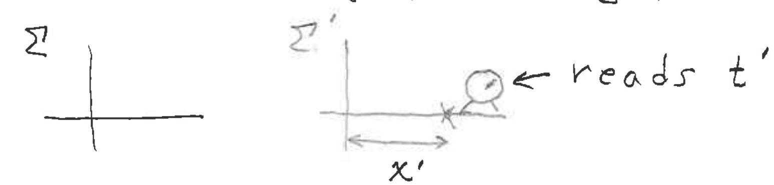


Figure out how to relate the coordinate (x, y, z, t) & (x', y', z', t') of the same event E:

- $y = y'$ and $z = z'$ since the same line in space is specified by each pair
- given x' & t' find x & t:



$$t = \underbrace{\gamma t'}_{\text{if } x'=0} + \underbrace{\frac{\gamma x' v}{c^2}}$$

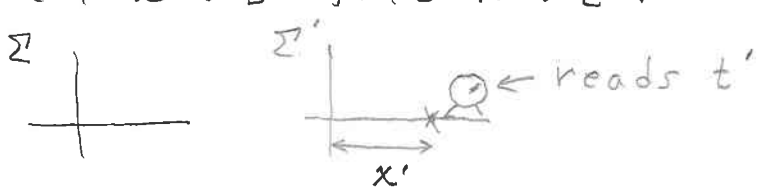
When Σ's clocks read t, clock at origin of Σ' reads t'

time is later by this amount because down stream actually occurs later

$$t = \gamma\left(t' + \frac{v}{c^2} x'\right)$$

Figure out how to relate the coordinate (x, y, z, t) & (x', y', z', t') of the same event E:

- $y = y'$ and $z = z'$ since the same line in space is specified by each pair
- given x' & t' find x & t:



$$t = \underbrace{\underbrace{\gamma t'}_{\text{if } x'=0}}_{\text{When } \Sigma\text{'s clocks read } t\text{, clock at origin of } \Sigma' \text{ reads } t'} + \underbrace{\frac{\gamma x' v}{c^2}}_{\text{time is later by this amount because down stream actually occurs later}}$$

$$t = \gamma\left(t' + \frac{v}{c^2}x'\right)$$

$$x = vt + x'\frac{1}{\gamma}$$

$$= v\left(\gamma t' + \cancel{\frac{v}{c^2}x'}\right) + \gamma x'\left(1 - \cancel{\frac{v^2}{c^2}}\right)$$

$$= \gamma(x' + vt')$$

Summary

event E has coordinates (x, y, z, t) in Σ & (x', y', z', t') in Σ'. These are related by a linear Lorentz transformation

$$x = \gamma(x' + vt')$$
$$y = y'$$
$$z = z'$$
$$t = \gamma\left(t' + \frac{v}{c^2}x'\right)$$

This transformation contains time dilation, length contraction and asynchronicity

$$x = vt + x'\frac{1}{\gamma}$$

$$= v\left(\gamma t' + \cancel{\frac{v}{c^2}x'}\right) + \gamma x'\cancel{\left(1 - \frac{v^2}{c^2}\right)}$$

$$= \gamma(x' + vt')$$

Summary

event E has coordinates (x, y, z, t) in Σ & (x', y', z', t') in Σ'. These are related by a linear Lorentz transformation

$$x = \gamma(x' + vt')$$

$$y = y'$$

$$z = z'$$

$$t = \gamma\left(t' + \frac{v}{c^2}x'\right)$$

This transformation contains time dilation, length contraction and asynchronicity

5. Demonstrate

a) Moving clock at origin of Σ'

two events:

$x_1' = 0,\ t_1'$

$x_2' = 0,\ t_1' + \Delta t'$

What does Σ think $t_1 = \gamma\left(t_1' + \frac{v}{c^2}x_1'\right)$

$$t_2 = \gamma\left(t_1' + \Delta t' + \frac{v}{c^2}x_1'\right)$$

∴ elapsed time for Σ:

$$\Delta t = t_2 - t_1 = \gamma \Delta t'$$

Σ' clock running slow by factor γ ✓

not $x_1 = \gamma(x_1' + vt_1')$

$$x_2 = \gamma\left[x_1' + v(t_1' + \Delta t')\right]$$

$$x_2 - x_1 = \gamma v \Delta t' = v\Delta t$$

A point fixed in Σ' is moving with velocity v ✓

5. Demonstrate

a) Moving clock at origin of Σ'

two events:

$x'_1 = 0, \ t'_1$

$x'_2 = 0, \ t'_1 + \Delta t'$

What does Σ think

$$t_1 = \gamma\left(t'_1 + \frac{v}{c^2} x'_1\right)$$

$$t_2 = \gamma\left(t'_1 + \Delta t' + \frac{v}{c^2} x'_1\right)$$

$\therefore$ elapsed time for Σ:

$$\Delta t = t_2 - t_1 = \gamma \Delta t'$$

Σ' clock running slow by factor γ ✓

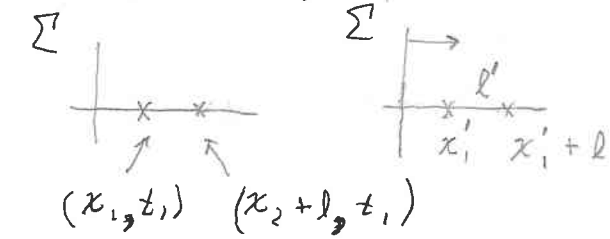
not $x_1 = \gamma(x'_1 + v t'_1)$

$$x_2 = \gamma\left[x'_1 + v(t'_1 + \Delta t')\right]$$

$$x_2 - x_1 = \gamma v \Delta t' = v \Delta t$$

A point fixed in Σ' is moving with velocity v ✓

b) Moving length l' fixed in Σ'



$$x'_1 = \gamma(x_1 - v t_1)$$

$$x'_2 = \gamma(x_1 + l - v t_1)$$

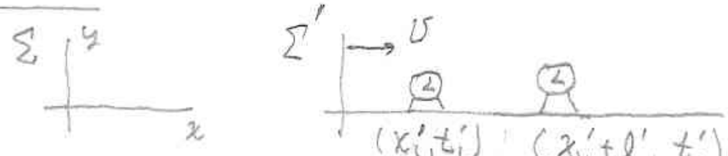
$l' = x'_2 - x'_1 = \gamma l$ or $l = l'/\gamma$ ✓

$t'_1 = \gamma\left(t_1 - \frac{v}{c^2} x_1\right)$ $\quad t'_2 = \gamma\left(t_1 - \frac{v}{c^2}(x_1 + l)\right)$

$$\therefore \ t'_2 - t'_1 = -\gamma l \frac{v}{c^2}$$

Σ' says Σ measure right end of moving l' to soon and therefore deduced too short a length!

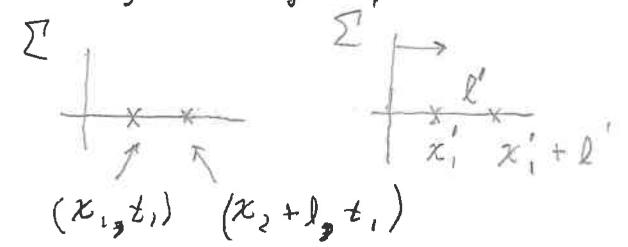
c)



$t_1 = \gamma\left(t'_1 + \frac{v}{c^2} x'_1\right)$ $\quad t_2 = \gamma\left(t'_1 + \frac{v}{c^2}(x'_1 + l')\right)$

$$t_2 - t_1 = \gamma \frac{v}{c^2} l'$$ ✓

b) Moving length, l' fixed in Σ'



$$x_1' = \gamma(x_1 - v t_1)$$

$$x_2' = \gamma(x_1 + l - v t_1)$$

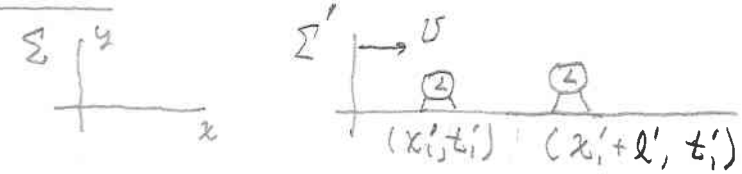
$$l' = x_2' - x_1' = \gamma l \quad \text{or} \quad l = l'/\gamma \quad ✓$$

$$t_1' = \gamma\left(t_1 - \frac{v}{c^2} x_1\right) \qquad t_2' = \gamma\left(t_1 - \frac{v}{c^2}(x_1 + l)\right)$$

$$\therefore \quad t_2' - t_1' = -\gamma l \frac{v}{c^2}$$

Σ' says Σ measure right end of moving l' to soon and therefore deduced too short a length!

c)



$$t_1 = \gamma\left(t_1' + \frac{v}{c^2} x_1'\right) \qquad t_2 = \gamma\left(t_1' + \frac{v}{c^2}(x_1' + l')\right)$$

$$t_2 - t_1 = \gamma \frac{v}{c^2} l' \quad ✓$$

6. Easy to transform velocities

Look at two events in Σ' along the particles trajectory:

$$(x_1', t_1') \quad \& \quad (x_1' + u'(t_2' - t_1'),\ t_2')$$

$$x_1 = \gamma(x_1' + v t_1') \qquad x_2 = \gamma[x_1' + u'(t_2' - t_1') + v t_2']$$

$$t_1 = \gamma\left(t_1' + \frac{v}{c^2} x_1'\right) \qquad t_2 = \gamma\left[t_2' + \frac{v}{c^2}(x_1' + u'(t_2' - t_1'))\right]$$

$$u = \frac{x_2 - x_1}{t_2 - t_1} = \frac{\gamma[u'(t_2' - t_1') + v(t_2' - t')]}{\gamma\left[t_2' - t_1' + \frac{vu'}{c^2}(t_2' - t_1')\right]}$$

or

$$u = \frac{u' + v}{1 + \frac{vu'}{c^2}}$$

as $u' \to c$ $\quad u \to \dfrac{c + v}{1 + \frac{v}{c}} = c$

as $v \to c$ $\quad u \to \dfrac{u' + c}{1 + \frac{u'}{c}} = c$