③ a and b time-like

$$0 > (a-b)^2 = (\Delta x')^2 - c^2(\Delta t)^2$$

$$|\Delta t| > \frac{1}{c}|\Delta x|$$

enough time for a signal slower than light to reach b from a

$$c\Delta t = \gamma(c|\Delta t'| + \beta|\Delta x'|)$$

$\beta|\Delta x'|$ term cannot change time order, a & b can be causally connected

① ACME acausal alarm company sees robbery underway at 1 AM

② send superluminal signal which reaches Police before 1 AM

③ quick acting police send super-luminal signal to patrol car.

④ robber arrested before crime

November 5, 2020

10. Doppler effect for light

Σ labels these two events

$$x_1 = \gamma(x' + vt') \qquad t_1 = \gamma(t' + \frac{v}{c^2}x')$$

$$x_2 = \gamma(x' + \lambda' + vt') \qquad t_2 = \gamma(t' + \frac{v}{c^2}(x'+\lambda'))$$

$$\lambda = x_2 - x_1 + \underbrace{c(t_2 - t_1)}$$

since wave moved to the left by this amount between two measurements

$$= \gamma\lambda' + c\gamma\frac{v}{c^2}\lambda' = \frac{1}{\sqrt{1-v^2/c^2}}\lambda'(1+\frac{v}{c})$$

$$= \sqrt{\frac{1+v/c}{1-v/c}}\,\lambda'$$

wave length of light from receding star is longer, $\lambda\nu = c \Rightarrow \nu$ smaller

explains "red shift

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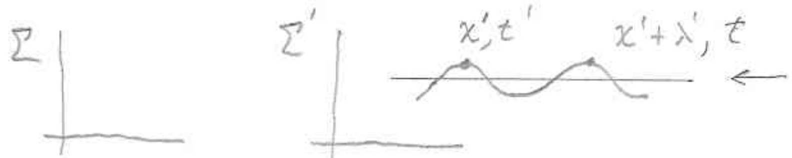

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C Dynamics

1. Define momenta

$$\vec{p} = \lim_{\Delta t \to 0} m \frac{\Delta\vec{r}}{dt}$$

} complicated unnatural ratio of four-vector components

Beter to use a "universal" scalar time in the denominator. Use the "proper time" the time read on a clock moving with the particle

If particle is moving with velocity v in Σ and a proper time interval of elapsed time $\Delta\tau$ has passed, Σ will say $\Delta t = \gamma_v \Delta\tau = \frac{1}{\sqrt{1-v^2/c^2}}\Delta\tau$

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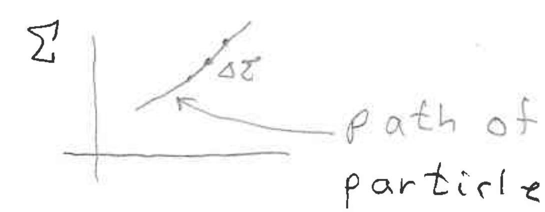

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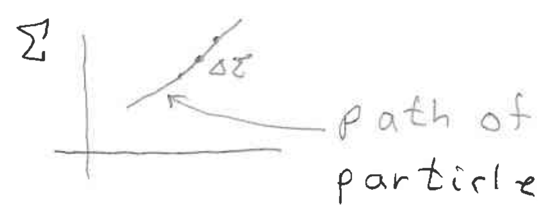

$\therefore \quad \frac{\Delta\tau}{\Delta t} = \frac{1}{\gamma_v}$ or $\frac{d\tau}{dt} = \sqrt{1 - \left(\frac{|v(t)|}{c}\right)^2}$

or $\tau_2 - \tau_1 = \int_{t_1}^{t_2} \sqrt{1 - \frac{v^2(t)}{c^2}}\, dt$

Note τ is a scalar quantity, the same in all systems.

Try $\vec{p} = \lim_{\Delta t \to 0} m \frac{\vec{r}(t+\Delta t) - \vec{r}(t)}{\Delta\tau}$

$$= \lim_{\Delta t \to 0} m \frac{\vec{r}(t+\Delta t) - \vec{r}(t)}{\Delta t} \gamma_v = m\gamma_v \vec{v}$$

Just old Newtonian momentum $\times \gamma_v$

Boring? No! $\vec{r}(t+\Delta t) - \vec{r}(t)$ are 3 components of a four-vector.

What is the fourth component?

$\vec{p} = m\gamma\vec{v} \quad p_4 = \lim_{\Delta t \to 0} \frac{c(t_2 - t_1)}{\Delta t} m\gamma_v$

$$= mc\gamma_v = mc \frac{1}{\sqrt{1-v^2/c^2}}$$

for $\frac{v}{c} \ll 1 \quad \approx mc\left[1 + \frac{1}{2}\frac{v^2}{c^2}\right] = \frac{mc^2 + \frac{1}{2}mv^2}{c} = \frac{E}{c}$

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This does describe relativistic motion correctly. Energy E and momentum $\vec{p}$ combine into a single four-vector

$$p = \left(p_1, p_2, p_3, \frac{E}{c}\right)$$

$$\vec{p} = m\gamma_v \frac{d\vec{r}}{dt} \qquad E = mc^2\gamma_v$$

Now if momentum is conserved in all intertial systems energy must be too

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$+10^4 \frac{m}{sec}$ $\quad$ 1KG $\longrightarrow$ $\quad$ M $= ?$ $\quad$ $\longleftarrow$ 1 kg $\quad$ $-10^4 \frac{m}{sec}$

$$2\left[Mc^2 + \frac{1}{2}Mv^2\right] = M'c^2$$

$$M' - 2M = M\frac{v^2}{c^2} = 1\,\mathrm{KG} \times \left[\frac{10^4}{3\times 10^8}\right]^2$$

$$\approx 10^{-9}\,\mathrm{KG} \sim 1\,\mu g$$

Useful relations:

If $\vec{p} = m\vec{v}\gamma_v$ & $E/c = mc\gamma_v$ are the components of a four-vector what is its invariant length?

$$\vec{p}^2 - \left(\frac{E}{c}\right)^2 = m^2\vec{v}^2\gamma_v^2 - m^2c^2\gamma_v^2$$

$$= m^2c^2\left[\frac{v^2}{c^2} - 1\right]\frac{1}{1 - v^2/c^2} = -m^2c^2$$

Thus $p^2 = -m^2c^2$ and m is a scalar which was our starting hypothesis