4. Recall result for potential energy

for gravity $\quad U(\vec{r}_1, \vec{r}_2) = -\dfrac{G m_1 m_2}{|\vec{r}_1 - \vec{r}_2|}$

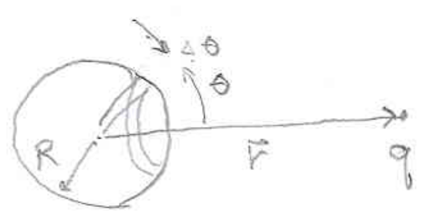
$$\vec{F}_{1\ \text{on}\ 2} = -\vec{\nabla}_{r_2}\left[-\frac{G m_1 m_2}{\sqrt{(x_1-x_2)^2+(y_1-y_2)^2+(z_1-z_2)^2}}\right]$$

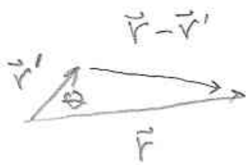
$$= -\frac{G m_1 m_2 (\vec{r}_2 - \vec{r}_1)}{|\vec{r}_1 - \vec{r}_2|^3} \quad \checkmark$$

for two charges $\quad U(r_1, r_2) = +\dfrac{q_1 q_2}{|\vec{r}_1 - \vec{r}_2|}$

for N charges $\quad U(\vec{r}_1, \dots \vec{r}_N) = \dfrac{1}{2}\sum_{i \neq j} \dfrac{q_i q_j}{|\vec{r}_i - \vec{r}_j|}$

5. Find the potential energy of a charge $q$ displaced by $\vec{r}$ from the center of a sphere (not a ball) of charge $Q$ with radius $R$




$$|r - r'|^2 = r^2 + R^2 - 2rR\cos\theta$$

December 3, 2020



Add up the potential energy of each ring of charge, equidistant from $q$:

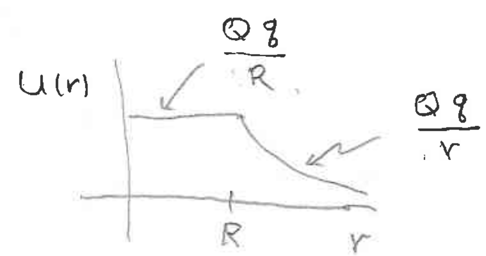
$$U(r) = \int_0^\pi \underbrace{d\theta\, R\ 2\pi R \sin\theta}_{\text{area of ring}}\ \underbrace{\frac{Q}{4\pi R^2}}_{\substack{\text{surface density}\\ \text{of charge}\\ \text{on sphere}}}\ \frac{q}{\sqrt{R^2+r^2-2Rr\cos\theta}}$$

let $z = \cos\theta$

$$= \frac{Qq}{2}\int_{-1}^{1} dz\, \frac{1}{\sqrt{R^2+r^2-2Rrz}}$$

$$= \frac{Qq}{2}\left(-\frac{1}{2Rz}\right) 2\Big\{\underbrace{\underbrace{\sqrt{R^2+r^2-2Rr}}_{|R-r|} - \underbrace{\sqrt{R^2+r^2+2Rr}}_{|R+r|}}_{-2\times\text{smaller}(R,r)}\Big\}$$

$$= Qq\,\frac{1}{\text{larger}(R,r)}$$

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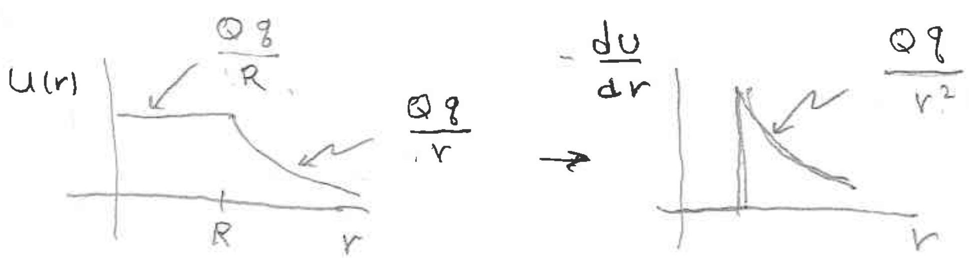




Outside sphere, $r > R$

force from sphere of charge $Q$ is the same as from a point charge $Q$ at the center of the sphere

Inside sphere, $r < R$

force from sphere is zero

Now we can easily answer the question: "How much energy is needed to assemble a sphere of charge $Q$ of radius $R$?"

$$\Delta U = \frac{Q \cdot \Delta Q}{R}$$

is increase in potential energy needed to bring $\Delta Q$ in from infinity to the radius $R$ of the sphere.

Easily integrate

$$U = \int_0^U dU = \int_0^Q \frac{Q\,dQ}{R} = \frac{1}{2}\frac{Q^2}{R}$$

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Second derivation:

$$U = \frac{1}{2}\sum_{i \neq j} \frac{\delta q_i\, \delta q_j}{|\vec{r}_i - \vec{r}_j|} \qquad Q = \sum_i \delta q_i$$

First sum over $j$ with $\vec{r}_i$ held fixed:

$$\sum_j \frac{\delta q_i\, \delta q_j}{|\vec{r}_i - \vec{r}_j|} = \delta q_i \frac{Q}{R}$$

energy of $\delta q_i$ at sphere surface

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---

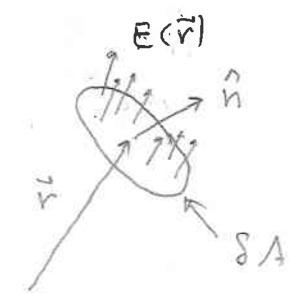
B Gauss' law

1. Flux $\quad \vec{E}(\vec{r})$ is called a vector field since it determines the vector $\vec{E}$ at every position $\vec{r}$.

The flux of $\vec{E}(\vec{r})$ through a small area $\delta A$ at the position $\vec{r}$ with the unit normal vector $\hat{n}$ is

$$\text{Flux} = \delta A\, \hat{n} \cdot \vec{E}(\vec{r})$$

$$= \underbrace{\delta A \cos\theta}_{\text{perpendicular area}} |\vec{E}(\vec{r})|$$



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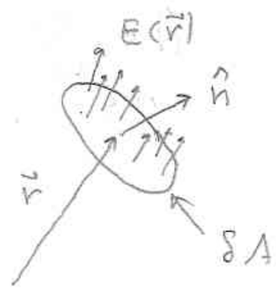




The concept of flux of electric field is important but abstract

Consider a more familiar analogy:

a fluid of mass density $\rho(\vec{r})$ moving with velocity $\vec{v}(\vec{r})$.

Define $\vec{p}(\vec{r}) = \rho(\vec{r})\,\vec{v}(\vec{r})$ as the density of momentum at $\vec{r}$, also a vector field. For a small volume $\delta V$

$$\vec{p}(\vec{r})\,\delta V = \vec{v} \underbrace{\rho\,\delta V}_{\text{mass in } \delta V} = \text{momentum of mass in } \delta V$$

The flux of $\vec{p}(\vec{r})$ through $\delta A$ is the rate at which mass flows thru $\delta A$


volume $(\Delta t\, v)\,\delta A \cos\theta$ moves thru $\delta A$ in time $\delta t$

rate mass flows thru $\delta A$

$$= \frac{\rho\,[\Delta t\, v\, \delta A \cos\theta]}{\Delta t}$$

$$= \rho(\vec{r})\, v(\vec{r})\, \delta A \cos\theta$$

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2. Gauss' law:

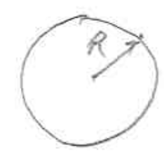
Flux of $\vec{E}(\vec{r})$ thru a closed surface $S$ which is the boundary of a volume $V$ $(S = \partial V)$ is $4\pi Q$ where $Q$ is the charge in $V$

$$\int_S \vec{E}(\vec{r})\cdot\hat{n}\, S = 4\pi Q_{\text{enclosed}}$$

$$= 4\pi \int_V d^3r\ \rho(\vec{r})$$

Proof:

a) For a point charge $q$ at the center of a sphere of radius $R$




$$\vec{E}(\vec{r}) = + \frac{\vec{r}}{r^3}\, q$$

Flux thru sphere -

$$= \int ds\ \hat{n}\cdot\vec{E}(\vec{r}) = \int ds\ \frac{q}{R^2}$$

$$= (4\pi R^2)\cdot \frac{q}{R^2}$$

$$= q \quad \text{as claimed}$$

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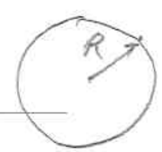


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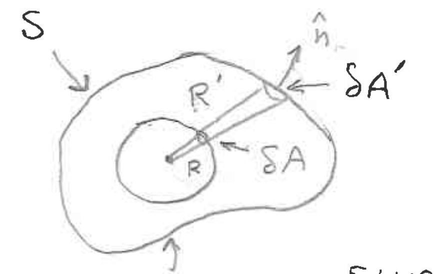
b) For a point charge q inside a general surface



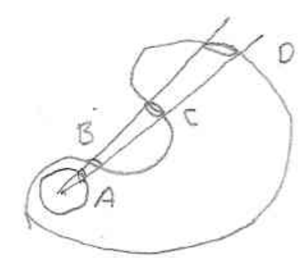

If $\delta A$ on sphere of radius R is enclosed in cone, and $\delta A'$ on surface S is enclosed in cone, then perpendicular area

$$\delta A' \cancel{\cos\theta} = \frac{R'^2}{R^2} \delta A$$ making flux of $\vec{E}(r)$ thru $\delta A'$ & $\delta A$ equal

$$\underbrace{\delta A' \cos\theta}_{\frac{R'^2}{R^2}\delta A} \frac{q^2}{R'^2} \stackrel{?}{=} \delta A \frac{q^2}{R^2}$$

$$\delta A \frac{q^2}{R^2} \stackrel{\checkmark}{=} \frac{q^2}{R^2} \delta A$$

c) Works for complex cases too

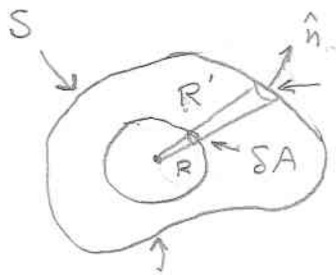



$$(\text{Flux})_A = (\text{Flux})_B + (\text{Flux})_C + (\text{Flux})_D$$

$+ \quad + \quad -$

[Of course find zero flux if q lies outside of S]

b) For a point charge $q$ inside a general surface

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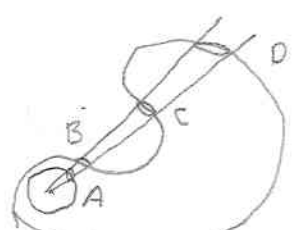


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d) Finally add N point charges together: $\vec{E}_i(\vec{r}) = \dfrac{q_i}{|\vec{r}-\vec{r}_i|^3}(\vec{r}-\vec{r}_i)$


$$\int_S \vec{E}_i(\vec{r})\cdot\hat{n}\, dS = 4\pi q_i$$

$$\Rightarrow \int_S \underbrace{\sum_{i=1}^{N} E_i(\vec{r})}_{\vec{E}(\vec{r})} \cdot \hat{n}\, dS = 4\pi \sum_{i=1}^{N} q_i$$

$$\text{or } \int_{S=\partial V} \vec{E}(\vec{r})\cdot\hat{n}\, dS = 4\pi \int_V \rho(r)\, d^3r$$
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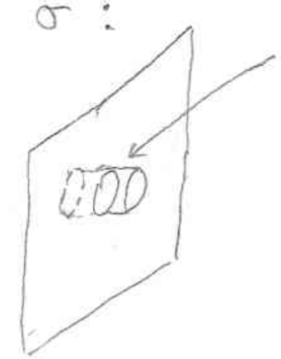
3. Use Gauss' law and symmetry to solve many problems.

a) Find $\vec{E}(\vec{r})$ produced by an infinite plane of charge with a uniform charge density $\sigma$:



Gaussian surface S perpendicular surface with faces of area A

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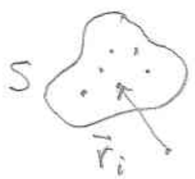


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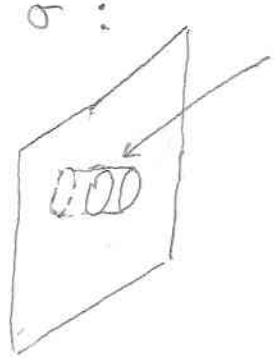
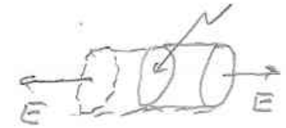


Gaussian surface S perpendicular surface with faces of area A



Symmetry of problem requires $\vec{E}$ $\perp$ to plane and out or in on both sides

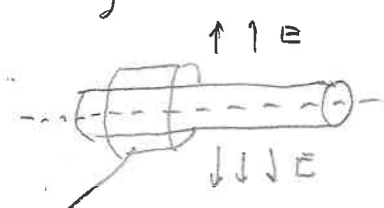



$$\int_S \vec{E}\cdot\hat{n}\,dS = 4\pi A\sigma$$

$$2AE$$

or $E(\vec{r}) = 2\pi\sigma$, out for $\sigma > 0$, in for $\sigma < 0$

b) Find $\vec{E}(\vec{r})$ for a long cylinder of charge of radius R an uniform surface charge density $\lambda$ per unit length




End View


Use Gaussian surface S of length L & radius R

$$\int_S \vec{E}\cdot\hat{n}\,dS = 4\pi Q$$

$$E\,2\pi r L \qquad 4\pi\lambda L$$

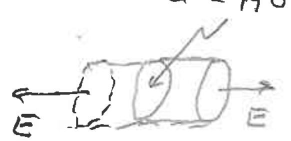
R

$$E(r) = \begin{cases} 2\lambda/r & r > R \\ 0 & r < R \end{cases}$$

$\perp$ to symmetry axis

Symmetry of problem requires $\vec{E} \perp$ to plane and out or in on both sides

$Q = A\sigma$

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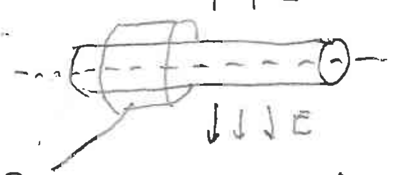


End view


Use Gaussian surface $S$ of length $L$ & radius $R$

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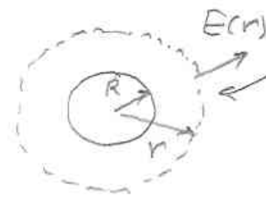
c) Finally reproduce result for the surface of a sphere containing a uniformly distributed charge $Q$:




Gaussian surface of radius $r$

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$$4\pi r^2 E(r) = 4\pi Q$$

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---

C Gauss' theorem

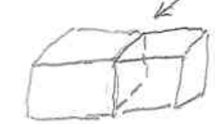
critical idea:

If a volume $V$ is expressed as a union of smaller volumes $\delta V_i$

$$V = \bigcup_{i=1}^{N} \delta V_i$$

$$\int_{\partial V} \vec{E}\cdot\hat{n}\,dS = \sum_{i=1}^{N} \int_{\partial(\delta V_i)} \vec{E}\cdot\hat{n}\,dS_i$$

Since fluxes through internal surfaces cancel



normals to this surface are in opposite directions!

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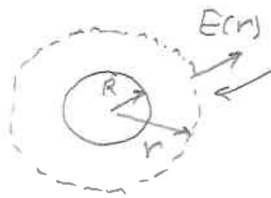


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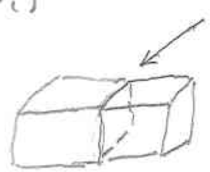

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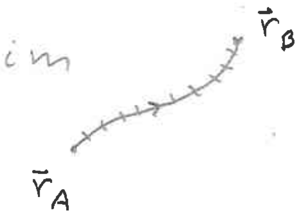
Now three cases

1-dim




$$V(r_B) - V(r_A) = \underbrace{\sum_{i=1}^{N}\left[V(r_{i+1}) - V(r_i)\right]}$$

$$\vec{r}_B = \vec{r}_{N+1}$$
$$\vec{r}_A = \vec{r}_1$$

$$\approx \sum_i \vec{\nabla} V\cdot(\vec{r}_{i+1} - \vec{r}_i)$$

$$\rightarrow \int_{r_A}^{r_B} \vec{\nabla}\cdot V(\vec{r})\cdot d\vec{r}$$

---

2-dim


$$S = \bigcup_{i=1}^{N} \delta S_i$$

$$\oint_{\partial S} \vec{F}\cdot d\vec{r} = \sum_{i=1}^{N} \oint_{\partial S_i} \vec{F}\cdot dr$$

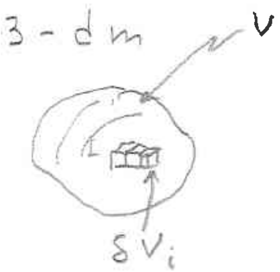
$$\approx \sum_{i=1}^{N} (\vec{\nabla}\times\vec{F})\cdot\hat{n}\,\delta S_i$$

$$\rightarrow \int_S (\vec{\nabla}\times\vec{F})\cdot\hat{n}\, dS$$

Stokes theorem

---

3-dm




$$V = \bigcup_i \delta V_i$$

$$\oint \vec{E}\cdot\hat{n}\, dS = \sum_{i=1}^{N} \int_{\partial V_i} \vec{E}(\vec{r})\cdot\hat{n}\, dS$$

$$\approx \sum_{i=1}^{N} (\vec{\nabla}\cdot\vec{E})\,\delta V_i$$

$$\rightarrow \int_V (\vec{\nabla}\cdot\vec{E})\, dV$$

Gauss' theorem

Now three cases 

1-dim


$$V(r_B) - V(r_A) = \underbrace{\sum_{i=1}^{N} \left[ V(r_{i+1}) - V(r_i) \right]}$$

$$\vec{r}_B = \vec{r}_{N+1}$$
$$\vec{r}_A = \vec{r}_1$$

$$\approx \sum_i \vec{\nabla} V \cdot (\vec{r}_{i+1} - \vec{r}_i)$$

$$\to \int_{r_A}^{r_B} \vec{\nabla} \cdot V(\vec{r}) \cdot d\vec{r}$$

---

2-dim


$$S = \bigcup_{i=1}^{N} \delta S_i$$

$$\oint_{\partial S} \vec{F} \cdot d\vec{r} = \sum_{i=1}^{N} \oint_{\partial S_i} \vec{F} \cdot dr$$

$$\approx \sum_{i=1}^{N} (\vec{\nabla} \times \vec{F}) \cdot \hat{n} \, \delta S_i$$

$$\to \int_S (\vec{\nabla} \times \vec{F}) \cdot \hat{n} \, ds$$

Stokes theorem

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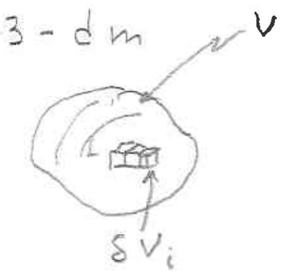
3-dm




$$V = \bigcup_i \delta V_i$$

$$\oint \vec{E} \cdot \hat{n} \, ds = \sum_{i=1}^{N} \int_{\partial V_i} \vec{E}(\vec{r}) \cdot \hat{n} \, ds$$

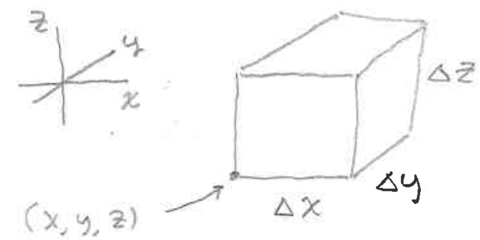
$$\approx \sum_{i=1}^{N} (\vec{\nabla} \cdot \vec{E}) \, \delta V_i$$

$$\to \int_V (\vec{\nabla} \cdot \vec{E}) \, dV$$

Gauss' theorem



must exploit small size of cube $\delta V_i$ to introduce derivative




$$\int \vec{E} \cdot \hat{n} \, ds$$

$$= \int_y^{y+\Delta y} dy' \int_z^{z+\Delta z} dz' \left[ E_x(x+\Delta x, y', z') - E_x(x, y', z') \right]$$

$$+ \int_x^{x+\Delta x} dx' \int_z^{z+\Delta z} dz' \left[ E_y(x', y+\Delta y, z') - E_y(x', y, z') \right]$$

$$+ \int_x^{x+\Delta x} dx' \int_y^{y+\Delta y} dy' \left[ \underbrace{E_z(x', y', z+\Delta z) - E_z(x', y', z')}_{\approx \frac{\partial E_z}{\partial z}(x', y', z) \Delta z} \right]$$

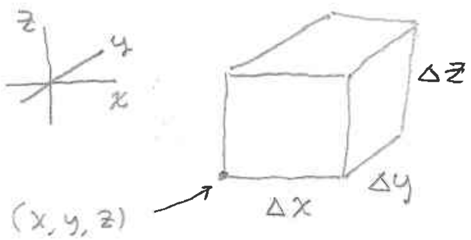
$$\approx \Delta y \Delta z \left[ \frac{\partial E_x}{\partial x}(x, y, z) \Delta x \right] + \Delta x \Delta z \left[ \frac{\partial E_y}{\partial y} \Delta y \right]$$
$$+ \Delta x \Delta y \left[ \frac{\partial E_z}{\partial z}(x, y, z) \Delta z \right]$$

$$= \Delta x \Delta y \Delta z \left[ \underbrace{\frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z}} \right]$$

divergence of $\vec{E}$ $\longrightarrow$ $= \vec{\nabla} \cdot E$

$$= \Delta V \; \vec{\nabla} \cdot \vec{E}$$

must exploit small size of cube $\delta V_i$ to introduce derivative




$$\int \vec{E} \cdot \hat{n}\, ds$$

$$= \int_y^{y+\Delta y} dy' \int_z^{z+\Delta z} dz' \left[ E_x(x+\Delta x, y', z') - E_x(x, y', z') \right]$$

$$+ \int_x^{x+\Delta x} dx' \int_z^{z+\Delta z} dz' \left[ E_y(x', y+\Delta y, z') - E_y(x', y, z') \right]$$

$$+ \int_x^{x+\Delta x} dx' \int_y^{y+\Delta y} dy' \left[ \underbrace{E_z(x', y', z+\Delta z) - E_z(x', y', z')}_{\approx \frac{\partial E_z}{\partial z}(x', y', z)\,\Delta z} \right]$$

$$\approx \Delta y \Delta z \left[ \frac{\partial E_x}{\partial x}(x, y, z)\, \Delta x \right] + \Delta x \Delta z \left[ \frac{\partial E_y}{\partial y} \Delta y \right] + \Delta x \Delta y \left[ \frac{\partial E_z}{\partial z}(x, y, z)\, \Delta z \right]$$

$$= \Delta x \Delta y \Delta z \left[ \underbrace{\frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z}}_{} \right]$$

divergence of $\vec{E}$ $\longrightarrow$ $= \vec{\nabla} \cdot E$

$$= \Delta V \; \vec{\nabla} \cdot \vec{E}$$



Gauss' theorem

$$\int_{\partial V} \vec{E} \cdot \hat{n}\, ds = \int_V dV \,(\vec{\nabla} \cdot \vec{E})$$

Gauss' law

$$\int_{\partial V} \vec{E} \cdot \hat{n}\, ds = 4\pi \int dV\, \rho(\vec{r})$$

$\Rightarrow$ differential form of Coulomb's law

$$\vec{\nabla} \cdot \vec{E}(\vec{r}) = 4\pi\, \rho(\vec{r})$$