Now three cases

1-dim

$$V(r_B) - V(r_A) = \underbrace{\sum_{i=1}^{N} \left[V(r_{i+1}) - V(r_i) \right]}$$

$$\vec r_B = \vec r_{N+1} \qquad \vec r_A = \vec r_1$$

$$\approx \sum_i \vec\nabla V \cdot (\vec r_{i+1} - \vec r_i)$$

$$\rightarrow \int_{r_A}^{r_B} \vec\nabla \cdot V(\vec r) \cdot d\vec r$$

2-dim

$$S = \bigcup_{i=1}^{N} \delta S_i$$

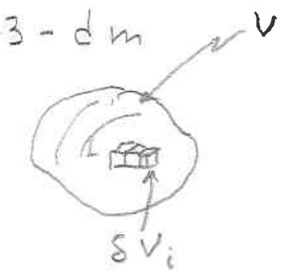
$$\oint_{\partial S} \vec F \cdot d\vec r = \sum_{i=1}^{N} \oint_{\partial S_i} \vec F \cdot d r$$

$$\approx \sum_{i=1}^{N} (\vec\nabla \times \vec F) \cdot \hat n \, \delta S_i$$

$$\rightarrow \int_S (\vec\nabla \times \vec F) \cdot \hat n \, ds$$

Stokes theorem

3-dim



$$V = \bigcup_i \delta V_i$$

$$\oint \vec E \cdot \hat n \, ds = \sum_{i=1}^{N} \int_{\partial V_i} \vec E(\vec r) \cdot \hat n \, ds$$

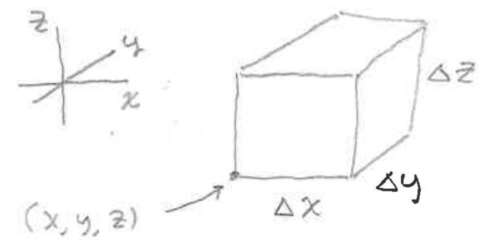
$$\approx \sum_{i=1}^{N} (\vec\nabla \cdot \vec E) \, \delta V_i$$

$$\rightarrow \int_V (\vec\nabla \cdot \vec E) \, dV$$

Gauss' theorem

December 8, 2020

must exploit small size of cube δV_i to introduce derivative



$$\int \vec E \cdot \hat n \, ds$$

$$= \int_y^{y+\Delta y} dy' \int_z^{z+\Delta z} dz' \left[E_x(x+\Delta x, y', z') - E_x(x, y', z') \right]$$

$$+ \int_x^{x+\Delta x} dx' \int_z^{z+\Delta z} dz' \left[E_y(x', y+\Delta y, z') - E_y(x', y, z') \right]$$

$$+ \int_x^{x+\Delta x} dx' \int_y^{y+\Delta y} dy' \left[\underbrace{E_z(x', y', z+\Delta z) - E_z(x', y', z')}_{\approx \frac{\partial E_z}{\partial z}(x', y', z) \Delta z} \right]$$

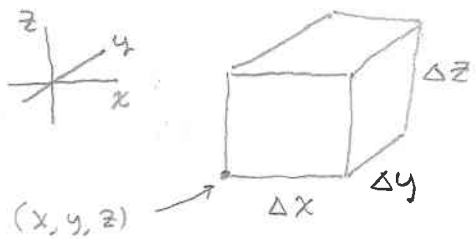
$$\approx \Delta y \Delta z \left[\frac{\partial E_x}{\partial x}(x, y, z) \Delta x \right] + \Delta x \Delta z \left[\frac{\partial E_y}{\partial y} \Delta y \right] + \Delta x \Delta y \left[\frac{\partial E_z}{\partial z}(x, y, z) \Delta z \right]$$

$$= \Delta x \Delta y \Delta z \left[\underbrace{\frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z}}_{} \right]$$

divergence of $\vec E$ $\longrightarrow$ $= \vec\nabla \cdot \vec E$

$$= \Delta V \; \vec\nabla \cdot \vec E$$

must exploit small size of cube δV_i to introduce derivative



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Gauss' theorem

$$\int_{\partial V} \vec{E}\cdot\hat{n}\, ds = \int_V dV\, (\vec{\nabla}\cdot\vec{E})$$

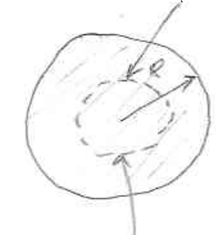
Gauss' law

$$\int_{\partial V} \vec{E}\cdot\hat{n}\, ds = 4\pi \int dV\, \rho(\vec{r})$$

$\Rightarrow$ differential (or local) form of Coulomb's law

$$\vec{\nabla}\cdot\vec{E}(\vec{r}) = 4\pi\,\rho(\vec{r})$$

Try an example: Consider a ball of uniform charge density ρ and radius R. Find $\vec{E}(\vec{r})$:



$$\int \vec{E}\cdot\hat{n}\, ds = 4\pi Q_{incl}$$

$$4\pi r^2 E(r) = 4\pi \rho \tfrac{4}{3}\pi r^3$$

$$E(r) = \begin{cases} \frac{4\pi}{3}\rho r & r<R \\ \frac{1}{r^2}\frac{4\pi}{3}\rho R^3 & r>R \end{cases}$$

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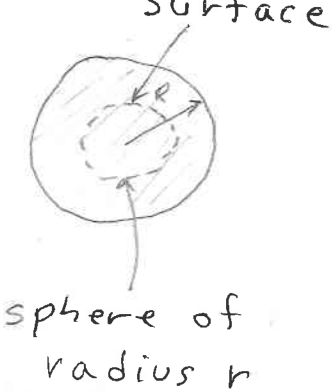

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Does $\vec{\nabla}\cdot\vec{E} = 4\pi\rho$?

$r>R$

$$\vec{\nabla}\cdot E = \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z}$$

$$= Q\left\{\frac{\partial}{\partial x}\left(\frac{x}{r^3}\right) + \frac{\partial}{\partial y}\left(\frac{y}{r^3}\right) + \frac{\partial}{\partial z}\left(\frac{z}{r^3}\right)\right\}$$

$$\frac{\partial}{\partial x}\frac{1}{(x^2+y^2+z^2)^{3/2}} = -\frac{3}{2}\times 2\frac{x}{(x^2+y^2+z^2)^{5/2}}$$

$$= Q\left\{\frac{1}{r^3}(1+1+1) - 3\frac{x^2+y^2+z^2}{(x^2+y^2+z^2)^{5/2}}\right\}$$

$$= 0 \checkmark$$

$r<R$

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Charge density $\rho(r)$ for a point charge

The Dirac delta function:

It is useful to define:

$$\lim_{R\to 0}\rho_R(r) = Q\,\delta^3(\vec{r}) \quad \text{where } \rho_R(r) = \begin{cases}\frac{Q}{\frac{4\pi R^3}{3}} & r<R \\ 0 & r>R\end{cases}$$

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$$= -\frac{3}{2}\times 2\frac{x}{(x^2+y^2+z^2)^{5/2}} \qquad \left. -3\frac{x^2+y^2+z^2}{(x^2+y^2+z^2)^{5/2}}\right\}$$

$$= 0 \checkmark$$

$r < R$

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$\delta^3(\vec{r})$ obeys two conditions:

① $\delta^3(\vec{r}) = 0$ for $\vec{r} \neq 0$

② $\int_{\delta V} \delta^3(\vec{r}) = 1$ if the point $\vec{r}=0$ lies in δV

Both conditions are obeyed by $\rho_R(r)$ in the limit $R\to 0$

$\delta^3(\vec{r})$ has many uses:

a) For a collection of point charges q_i at positions r_i we can write an explicit formula for the charge density

$$\rho(\vec{r}) = \sum_{i=1}^{N} q_i\,\delta^3(\vec{r}-\vec{r}_i)$$

b) We can write an equation for

$$\vec{\nabla}\cdot\left(\frac{\vec{r}}{r^3}\right) = \frac{3}{r^3} - 3\frac{\vec{r}^{\,2}}{r^5} = 4\pi\,\delta(\vec{r})$$

which is correct even at $\vec{r} = 0$

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D. Introduce the scalar potential $\Phi(\vec{r})$

For N charges q_i at r_i acting on an N+1st charge δq at $\vec{r}$

$$\vec{F}(\vec{r}) = \sum_{i=1}^{N} \frac{\vec{r} - \vec{r}_i}{|\vec{r} - \vec{r}_i|^3} \, \delta q \, q_i$$

is conservative so we can find a potential energy function $U(\vec{r})$ obeying

$$\vec{F}(\vec{r}) = -\vec{\nabla} U(\vec{r})$$

Can use

$$U(\vec{r}) = \sum_{i=1}^{N} \frac{q_i \, \delta q}{|\vec{r} - \vec{r}_i|}$$

Since $\vec{E}(\vec{r}) = \frac{1}{\delta q} \vec{F}(\vec{r})$

define $\Phi(\vec{r}) = \frac{1}{\delta q} U(r)$

$$\Rightarrow \quad \vec{E}(\vec{r}) = -\vec{\nabla} \Phi(\vec{r})$$

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Thus for N point charges

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note $\delta q\, \varphi(r)$ is not the total energy of all the charges but only that part which depends on $\vec{r}$.

Unit for potential

<u>esu</u> "stat volts" $= \frac{\text{esu}}{\text{cm}} = \frac{\sqrt{\text{dyne}}\cdot\text{cm}}{\text{cm}}$

$= \frac{\text{erg}}{\text{esu}} = \frac{\text{dyne cm}}{\sqrt{\text{dyne}}\ \text{cm}} = \sqrt{\text{dyne}}$

<u>SI</u> "Volt" $= \frac{\text{Joule}}{\text{Coulomb}} \equiv \frac{10^7\ \text{erg}}{3\times10^9\ \text{esu}}$

$= \frac{1}{300}$ statvolt

Units for electric field

<u>esu</u> $\frac{\text{statvolt}}{\text{cm}}$

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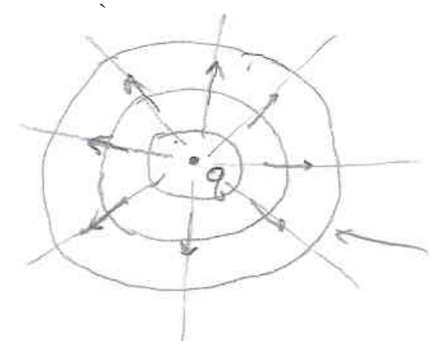
$\vec{E}(\vec{r})$ is a vector field proportion to the force on δq at $\vec{r}$

How can we visualize $\varphi(\vec{r})$?

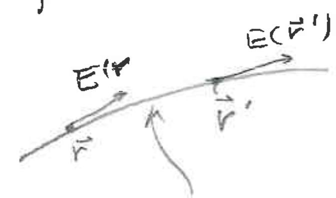
Draw surfaces of constant φ in 3-d. For a point charge

$\vec{E}(\vec{r}) = -\vec{\nabla}\varphi(r)$ is perpendicular to surface of constant φ



surface of constant φ

Draw "field lines" as curve $\vec{r}(\alpha)$ every where tangent to $E(\vec{r})$:



line $\vec{r}(\alpha)$

$$\frac{d\,\vec{r}(\alpha)}{dt} = \text{const}\cdot \vec{E}(\vec{r}(\alpha))$$

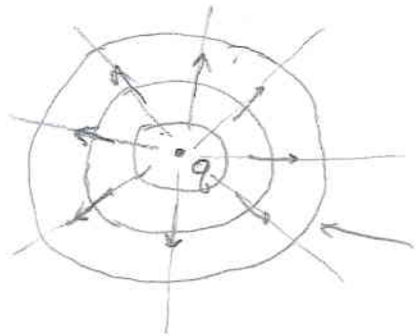
can use Euler's method given $\vec{E}(\vec{r})$ to integrate and find $\vec{r}(\alpha)$

$\vec{E}(\vec{r})$ is a vector field
proportion to the force on δq at $\vec{r}$

How can we visualize $\Phi(\vec{r})$?

Draw surfaces of constant Φ in 3-d. For a point charge

$\vec{E}(\vec{r}) = -\vec{\nabla}\Phi(r)$ is perpendicular to surface of constant Φ



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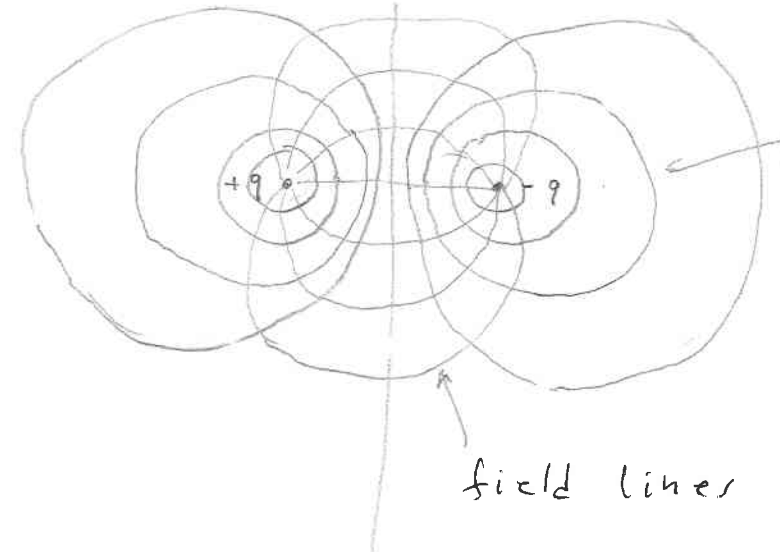
Most famous pattern of field lines and equipotential surfaces comes for two charges $\pm Q$



equipotential surfaces

field lines

With $\Phi(\vec{r})$ the local form of Gauss' law gives a new equation

$$\vec{\nabla}\cdot\vec{E} = 4\pi\rho$$

$$\vec{E} = -\vec{\nabla}\Phi$$

$$\vec{\nabla}\cdot(-\vec{\nabla}\Phi) = 4\pi\rho$$

or $$\vec{\nabla}^2\Phi = -4\pi\rho$$

$$\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$ the Laplacian

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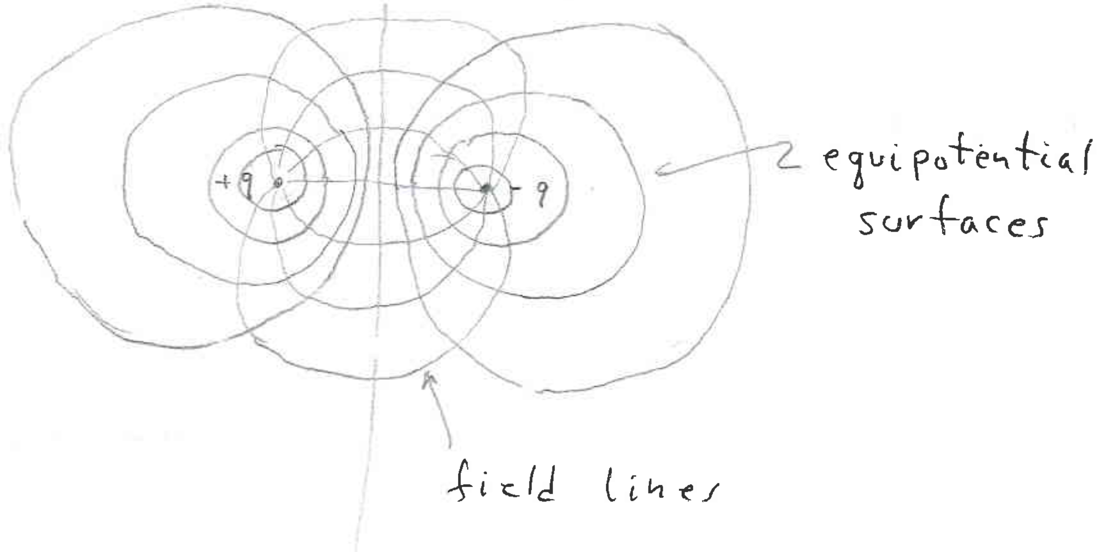

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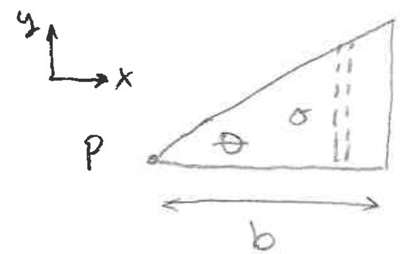
$$\vec{\nabla}^2\Phi = 4\pi\rho$$

is Poisson's equation

if $\rho = 0$ & $\vec{\nabla}^2\Phi = 0$ this is the Laplace equation.

Work out a problem from Purcell



Right triange of uniform charge density σ. Find the potential a P.

$$\Phi_P = \int_0^b dx \int_0^{x\tan\theta} dy \frac{\sigma}{\sqrt{x^2+y^2}}$$

$$\frac{1}{\sqrt{x^2+y^2}}$$

recall $$\frac{d}{dy}\ln\left[y+\sqrt{x^2+y^2}\right] = \underbrace{\frac{1}{y+\sqrt{x^2+y^2}}\left[1+\frac{y}{\sqrt{x^2+y^2}}\right]}$$

$$= \sigma\int_0^b dx \ln\left[\frac{x\tan\theta+\sqrt{x^2+x^2\tan^2\theta}}{x}\right]$$

$$= \sigma b \ln\left[\tan\theta + \underbrace{\sqrt{1+\tan^2\theta}}_{\frac{1}{\cos\theta}}\right]$$

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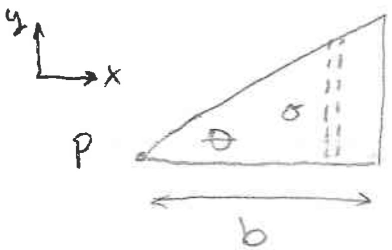

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$$= \sigma\int_0^b dx \, \ln\left[\frac{x\tan\theta + \sqrt{x^2+x^2\tan^2\theta}}{x}\right]$$

$$= \sigma b \ln\left[\tan\theta + \underbrace{\sqrt{1+\tan^2\theta}}_{\frac{1}{\cos\theta}}\right]$$

Use new math tools to express energy in terms of $\vec{E}(\vec{r})$

$$U = \frac{1}{2}\int_V d^3r \int d^3r' \, \rho(\vec{r})\frac{1}{|\vec{r}-\vec{r}'|}\rho(\vec{r}')$$

$$= \frac{1}{2}\int_V d^3r \, \underbrace{\rho(\vec{r})}_{\frac{1}{4\pi}\vec{\nabla}\cdot\vec{E}} \Big\{ \underbrace{\int d^3r' \frac{1}{|\vec{r}-\vec{r}'|}\rho(r')}_{\varphi(\vec{r})} \Big\}$$

$$= \frac{1}{8\pi}\int_V d^3r \left\{ \vec{\nabla}\cdot\left[\vec{E}(\vec{r})\,\varphi(\vec{r})\right] - \vec{E}(\vec{r})\cdot\vec{\nabla}\varphi \right\}$$

$$= \underbrace{\frac{1}{8\pi}\int_{\partial V} \hat{n}\cdot\left[\underbrace{\vec{E}(\vec{r})\,\varphi(\vec{r})}_{\sim\frac{1}{r^3}}\right] \underbrace{dS}_{r^2}}_{\text{Vanish as V grows}} + \frac{1}{8\pi}\int d^3r \, \vec{E}(\vec{r})^2$$

$$U = \int d^3r \left(\frac{\vec{E}(r)^2}{8\pi}\right)$$

energy density in the electric field

Use new math tools to express energy in terms of $\vec{E}(\vec{r})$

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energy density in the electric field

E Conductors

1. Electrical properties of materials

Recall all materials are made of atoms with a heavy compact positive charge nucleus surrounded by a much larger cloud of negatively charged electrons.

Two comon behaviors when $\vec{E}$ is applied:

insulator

a) All electrons a bound to a nucleus or cluster of nuclei and external $\vec{E}(\vec{r}) \neq 0$ does not make them move

conductor

b) One or more electron associated with some types of atoms present are accelerated by external $\vec{E} \neq 0$ and can flow from atom to atom in the direction of $-\vec{E}$

Use new math tools to express energy in terms of $\vec{E}(\vec{r})$

$$U = \frac{1}{2}\int_V d^3r \int d^3r' \; \rho(\vec{r}) \frac{1}{|\vec{r}-\vec{r}\,'|} \rho(\vec{r}\,')$$

$$= \frac{1}{2}\int_V d^3r \; \underbrace{\rho(\vec{r})}_{+\frac{1}{4\pi}\vec{\nabla}\cdot\vec{E}} \left\{ \underbrace{\int d^3r' \frac{1}{|\vec{r}-\vec{r}\,'|} \rho(r')}_{\Phi(\vec{r})} \right\}$$

$$= \frac{1}{8\pi}\int_V d^3r \left\{ \vec{\nabla}\cdot\left[\vec{E}(\vec{r})\,\Phi(\vec{r})\right] - \vec{E}(\vec{r})\cdot\vec{\nabla}\Phi \right\}$$

$$= \underbrace{\frac{1}{8\pi}\int_{\partial V} \hat{n}\cdot\left[\underbrace{\vec{E}(\vec{r})\,\Phi(\vec{r})}_{\sim \frac{1}{r^3}}\right] \underbrace{dS}_{r^2}}_{\text{vanish as V grows}} + \frac{1}{8\pi}\int d^3r \, \vec{E}(\vec{r})^2$$

$$U = \int d^3r \left(\frac{\vec{E}(r)^2}{8\pi}\right)$$

energy density in the electric field

Theory of electron motion

$$\vec{a}_e = -\frac{\vec{E}\,e}{m_e}, \qquad \vec{v}_e = -\frac{\vec{E}\,e}{m_e}\,t$$

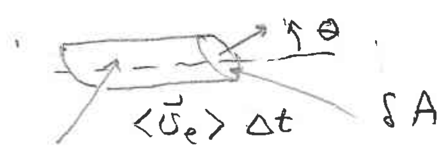
negative of electron's charge

but $\vec{v}_e$ does not increase without bound as t grows: the electron will collide with something in the material. Thus,

$$\langle \vec{v}_e \rangle = -\frac{\vec{E}\,e}{m_e} \overbrace{\langle t \rangle}^{\tau_{col}}$$

The average time between collisions

If there are n_e conduction electrons per unit volume the result is a flux or current of charge $\vec{j}(\vec{r}) = -e\,n_e \langle \vec{v}_e \rangle$



$$= \underbrace{\frac{e^2 \tau_{col} n_e}{m_e}}_{=\sigma} \vec{E}(\vec{r})$$

conductivity of material

charge thru δA per time

$$= \frac{-e n_e \langle \vec{v}_e \rangle \Delta t \, \delta A \cos\theta}{\Delta t} = \vec{j}\cdot\hat{n}\,\delta A$$

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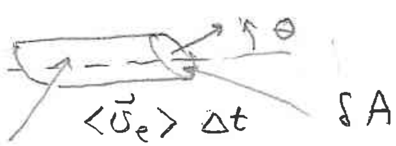

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$\vec{j}(\vec{r}) = \sigma\,\vec{E}(\vec{r})$ is Ohm's law

Typically the thermal velocity of the conduction electrons is much greater than the slow drift caused by $\vec{E}(\vec{r})$.

For electrostatics nothing should be moving so $\vec{j}(\vec{r}) = 0$ in a conductor which implies $\vec{E}(\vec{r}) = 0$