Physics UN2801 — Professor Christ

due 09/30/2021

Assignment #3

Reading:

Sept 28	Kleppner & Kolenkow 4.-4.10, 5.1-5.2
Sept 30	Kleppner & Kolenkow 5.3-5.8, Note 5.2

Problems:

18. Kleppner and Kolenkow 3.10
19. Kleppner and Kolenkow 3.17
20. Kleppner and Kolenkow 3.20
21. Find the 3×3 matrix M corresponding to a change of coordinate system from the basis $\hat{e}_1$, $\hat{e}_2$, $\hat{e}_3$ to a new basis $\hat{e}'_1$, $\hat{e}'_2$, $\hat{e}'_3$ by a rotation through θ about the $\hat{e}_2$ axis (clockwise as viewed along the direction of $\hat{e}_2$). What are the coordinates in the prime system of the vector $\vec{r} = 3\hat{e}_1 + 7\hat{e}_2 - 6\hat{e}_3$?
22. Consider a 3×3 matrix M: $M = \begin{pmatrix} M_{11} & M_{12} & M_{13} \\ M_{21} & M_{22} & M_{23} \\ M_{31} & M_{32} & M_{33} \end{pmatrix}$ and define the determinant of M by the formula:

$$\begin{aligned} \det M &= M_{11}M_{22}M_{33} + M_{12}M_{23}M_{31} + M_{13}M_{21}M_{32} \\ &- M_{13}M_{22}M_{31} - M_{12}M_{21}M_{33} - M_{11}M_{23}M_{32}. \end{aligned}$$

   (a) Show that $\det M = \det M^T$ where M^T is the transpose of M.

   (b) Using the fact that $\det(AB) = (\det A)(\det B)$ show that for an orthogonal matrix (*i.e.* $M^{-1} = M^T$), $\det M = \pm 1$.

   (c) Show that for the unit matrix: $I = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$, $\det I = +1$.

   (d) Find the matrix R that describes the change of basis vectors corresponding to the reflection: $\hat{e}'_1 = -\hat{e}_1$, $\hat{e}'_2 = -\hat{e}_2$, $\hat{e}'_3 = -\hat{e}_3$

   (e) Show that that $\det R = -1$.

   (f) Prove (perhaps using a continuity argument) that $\det M = +1$ for any matrix M describing a change of basis in which the new basis can be obtained from the original basis by performing a rotation of the original basis.

23. Kleppner and Kolenkow 4.4
24. Kleppner and Kolenkow 4.10
25. Kleppner and Kolenkow 4.13