Physics UN2802 — Professor Christ
due 03/18/2021

Assignment #18

Reading:

March 16	French and Taylor Chapter 2-1 thru 2-4
March 18	French and Taylor Chapter 2-5 thru 2-11

151. Purcell 8.15

152. Purcell 9.1

153. Purcell 9.5

154. French and Taylor 1-1

155. French and Taylor 1-3

156. French and Taylor 1-8

157. French and Taylor 1-10

158. Consider a 2-dimensional complex vector space with a orthonormal basis $\hat{e}_1$ and $\hat{e}_2$. Find the following:

   (a) The sum of the vectors $\psi + \psi'$ where $\psi = 3e^{i\pi/2}\hat{e}_1 + 2\hat{e}_2$ and $\psi' = 4\sqrt{2}e^{i\pi/4}\hat{e}_1 - 5\hat{e}_2$.

   (b) The three inner products: (ψ, ψ), (ψ', ψ') and (ψ', ψ).

   (c) Find unit vectors $\hat{\psi}_\perp$ and $\hat{\psi}'_\perp$ which are orthogonal to ψ and ψ' respectively.

159. Show that if a function $f(\phi)$ obeys

$$f(\phi_1)f(\phi_2) = f(\phi_1 + \phi_2) \tag{1}$$

   then $f(\phi)$ must have the form $f(\phi) = e^{c\phi}$ where c is a constant.

   [Hint: differentiate both sides of Eq. (1) with respect to ϕ_1, set $\phi_1 = 0$ and then solve the resulting equation.]

160. Find the two eigenvalues and corresponding eigenvectors for the 2×2 matrix

$$\begin{pmatrix} a & b \\ b & -a \end{pmatrix}$$

   where a and b are real numbers. Choose the eigenvectors to have unit length and show that they are orthogonal.

   [Hint: Let x and y be the two components of an eigenvector with eigenvalue λ. Solve the coupled equations

$$\begin{pmatrix} a & b \\ b & -a \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \lambda \begin{pmatrix} x \\ y \end{pmatrix}.$$

   to determine x, y and λ.]