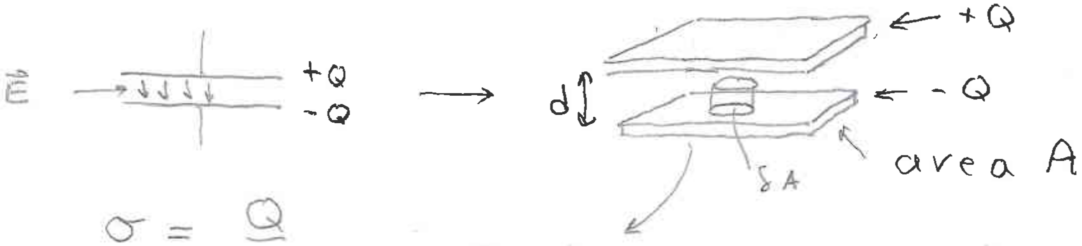
4. Parallel plate capacitor



$$\sigma = \frac{Q}{A}$$

$$E\,\delta A = -4\pi\sigma\,\delta A = -4\pi\frac{Q}{A}\,\delta A$$

$$\varphi_{top} - \varphi_{bot} \equiv V = -E\cdot d = \frac{4\pi Q}{A}d$$

$$\text{or}\quad V = \frac{1}{C}Q \quad \& \quad C = \frac{A}{4\pi d}$$

5. Units

for esu $\quad C = \dfrac{Q}{V} \sim \text{cm}$

for SI $\quad C = \dfrac{Q}{V} \sim \dfrac{\text{coulomb}}{\text{volt}} \equiv \text{Farad}$

$$1\ \text{Farad} = \frac{1\ \text{Coulomb}}{1\ \text{Volt}} = \frac{3\times 10^{9}\ \text{esu}}{\frac{1}{300}\ \text{stat-volt}} = 9\times 10^{11}\ \text{cm}$$

$$\uparrow (2.99792458)^2$$

a Farad is a large unit

a cm " " small unit

01/19/2021

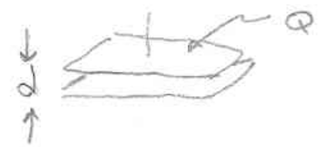
6. Energy stored in a capacitor

move δQ from lower to upper conductor

$$\delta W = V\,\delta Q = \frac{Q}{C}\,\delta Q$$

$$\mathcal{E} = \int_0^{\mathcal{E}} \delta W = \int_0^{Q}\frac{Q}{C}\,dQ = \frac{Q^2}{2C}$$

Check for case of // plate capacitor

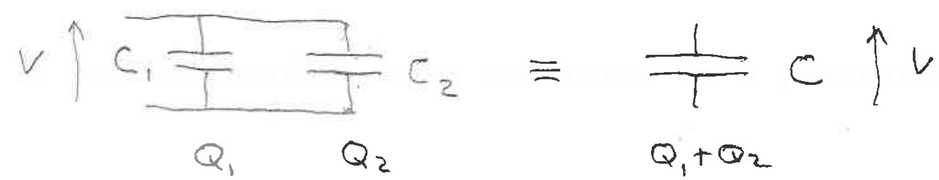


$$C = \frac{A}{4\pi d} \quad \& \quad \mathcal{E} = \frac{4\pi d}{2A}Q^2$$

$$\text{or}\quad \mathcal{E} = \underbrace{A\,d}_{\text{volume}}\ \underbrace{\frac{|\vec{E}|^2}{8\pi}}_{\text{energy density}} \qquad E = 4\pi Q/A$$

$$= A\,d\,\frac{(4\pi Q/A)^2}{8\pi} = 2\pi\frac{dQ^2}{A}\ \checkmark$$

7. Combine capacitors



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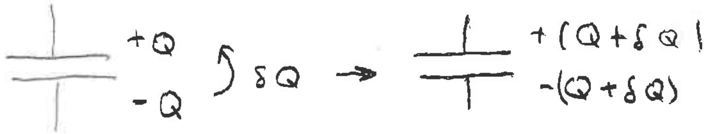

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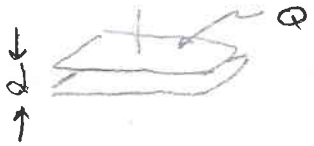

$$C = \frac{A}{4\pi d} \quad \Rightarrow \quad \mathcal{E} = \frac{4\pi d}{2A} Q^2$$

or $\mathcal{E} = \underbrace{Ad}_{\text{volume}} \underbrace{\frac{|\vec{E}|^2}{8\pi}}_{\text{energy density}}$. $\qquad E = 4\pi Q/A$

$$= Ad \frac{(4\pi Q/A)^2}{8\pi} = 2\pi \frac{dQ^2}{A} \quad ✓$$

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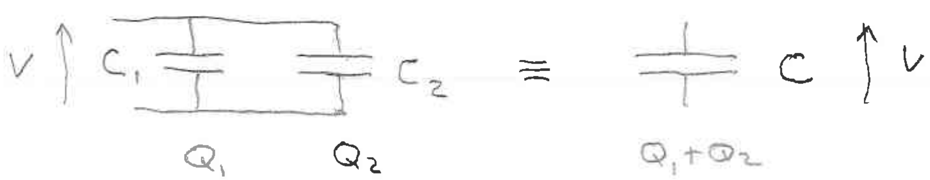
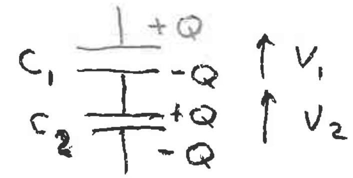


$$Q_1 = C_1 V, \quad Q_2 = C_2 V$$

So $\quad Q = Q_1 + Q_2 = \underbrace{(C_1 + C_2)}_{C} V$

When to capacitor are joined in parallel $\quad C = C_1 + C_2$



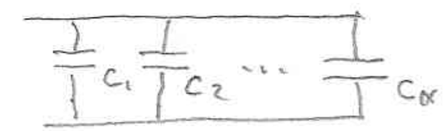
$$V = V_1 + V_2 = \frac{Q}{C_1} + \frac{Q}{C_2}$$

$$= \underbrace{\left(\frac{1}{C_1} + \frac{1}{C_2}\right)}_{1/C} Q$$

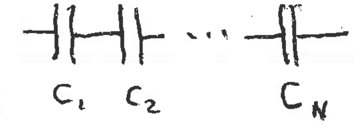
so $\quad C = \dfrac{1}{\frac{1}{C_1} + \frac{1}{C_2}} = \dfrac{C_1 C_2}{C_1 + C_2}$

Easy to generalize to the case of N capacitors:

parallel $\quad C = \sum_{i=1}^{N} C_i$



series: $\quad C = \dfrac{1}{\sum_{i=1}^{N} \frac{1}{C_i}}$



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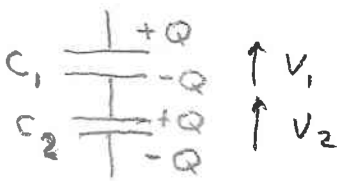

$$V = V_1 + V_2 = \frac{Q}{C_1} + \frac{Q}{C_2}$$

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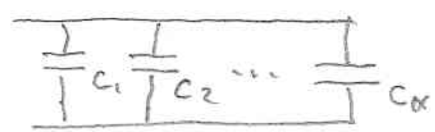

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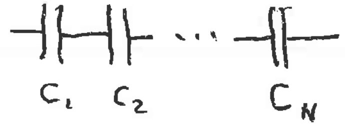
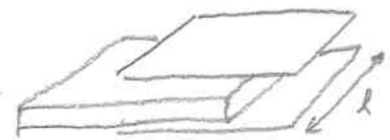
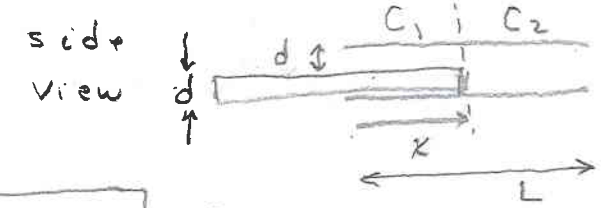
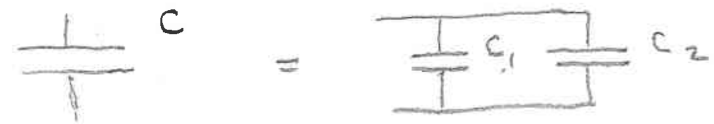


8. Problem. Find force on conductor sliding inside a parallel-plate capacitor







$C = C_1 + C_2$

$$C_1(x) = \frac{x\,l}{4\pi d} \qquad C_2(x) = \frac{(L-x)\,l}{4\pi \cdot 2d}$$

$$C(x) = C_1(x) + C_2(x) = \frac{l}{8\pi d}\left[2x + L - x\right]$$

$$= \frac{l}{8\pi d}\left[L + x\right]$$

$$U(x) = \frac{1}{2}\frac{Q^2}{C(x)} = \frac{1}{2} Q^2 \frac{8\pi d}{l} \frac{1}{L+x}$$

Inward force on conductor

$$F = -\frac{dU}{dx} = -\frac{4\pi d Q^2}{l} \frac{d}{dx} \frac{1}{L+x}$$

$$= \frac{4\pi d\, Q^2}{l (L+x)^2} \qquad \text{slab pulled in}$$

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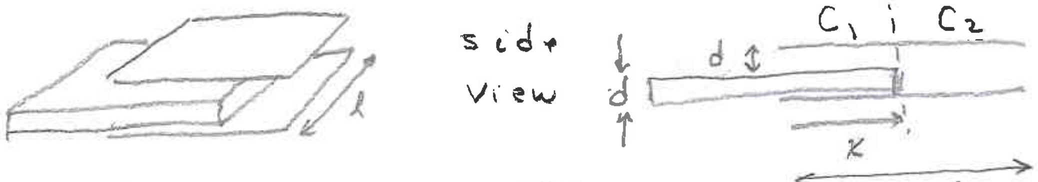

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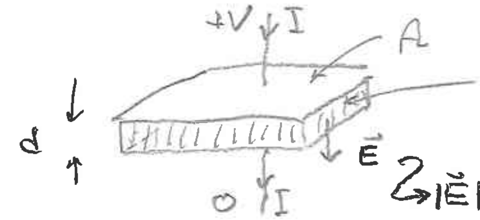
G Ohm's law and circuit theory

1. Resistors Recall in our model of conductivity $\underline{\vec{J} = \sigma \vec{E}}$

where $\vec{E}$ is the applied electric field, $\vec{J}$ the resulting current density esu/cm² sec

σ is the conductivity:



material with conductivity σ

$$|\vec{E}| = \frac{V}{d} \qquad \vec{J} = \sigma \cdot \vec{E}$$

$$\text{current } I = J A = \sigma \frac{V}{d} A$$

$$= \frac{V}{d/\sigma A} \equiv \frac{V}{R}$$

$V = IR$: Ohm's law $R = \dfrac{d}{\sigma A}$

Units esu: $\dfrac{\text{esu/cm}}{\text{esu/sec}} = \dfrac{\text{sec}}{\text{cm}}$

SI: $\dfrac{\text{Volt}}{\text{Amp}} \equiv \text{Ohm}$

$\dfrac{\text{Coulomb}}{\text{sec}}$ (Amp)

$$= \frac{(1/300)\ \text{stat volt}}{3\times10^{9}\ \text{esu/sec}}$$

$$= \frac{1}{9\times10^{11}}\ \frac{\text{sec}}{\text{cm}}$$

Thus $\dfrac{\text{sec}}{\text{cm}} = 9\times10^{11}$ Ohm

impractically large

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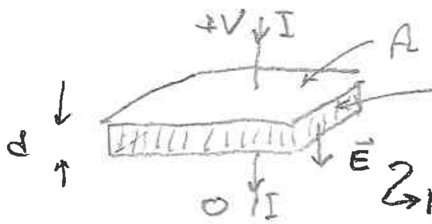

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$|\vec{E}| = \frac{V}{d}$ $\vec{J} = \sigma \cdot \vec{E}$

current $I = J A = \sigma \frac{V}{d} A$

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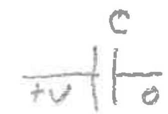
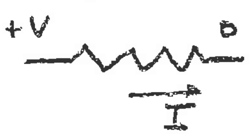
impractically large

2. Circuit theory

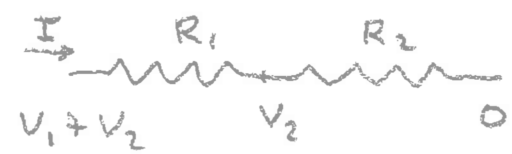
a) Introduce discrete elements with two terminal that can be connected with low (zero?) resistance wires
- easy to analyze
- Join to create more general function

two examples so far:

capacitor  $Q = VC$

resistor  $I = V/R$

b) Combine resistors



$I = \frac{V_1}{R_1} = \frac{V_2}{R_2}$

$\therefore V_1 + V_2 = (R_1 + R_2) I$

When combining resistors in series, add their resistances.

For N resistors in series $R = \sum_{i=1}^{N} R_i$

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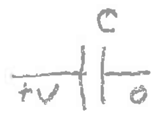
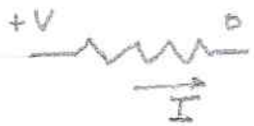
capacitor 

$Q = VC$

resistor 

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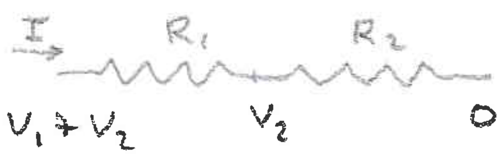

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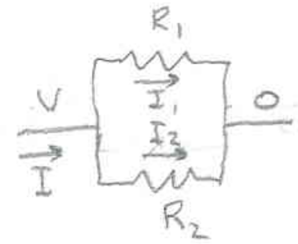

$$I = I_1 + I_2 = \frac{V}{R_1} + \frac{V}{R_2}$$

$$= \left(\frac{1}{R_1} + \frac{1}{R_2}\right) V = \frac{V}{\frac{1}{\frac{1}{R_1} + \frac{1}{R_2}}}$$

When combining two resistors in parallel, take the harmonic mean of their resistances:

$$R = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} = \frac{R_1 R_2}{R_1 + R_2}$$

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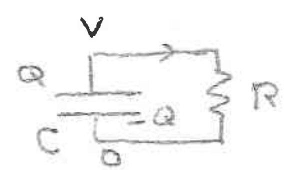
$$R = \frac{1}{\sum_{i=1}^{N} \frac{1}{R_i}}$$

c) Combine R + C

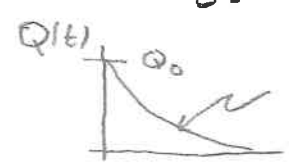


$V = Q/C \qquad I = V/R$

$$I = -\frac{dQ}{dt}$$

$$\therefore \quad \frac{dQ}{dt} = -\frac{V}{R} = -\frac{Q}{RC} \quad \text{or} \quad Q(t) = Q_0 e^{-\frac{t}{RC}}$$

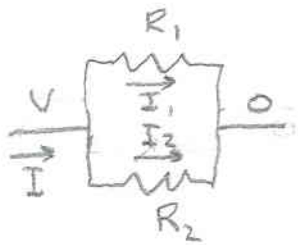
cm sec/cm



$Q_0 e^{-t/RC}$

$Q(t=0) = Q_0$

an "RC" circuit



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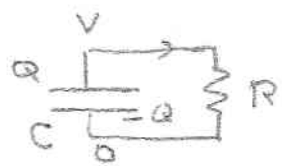

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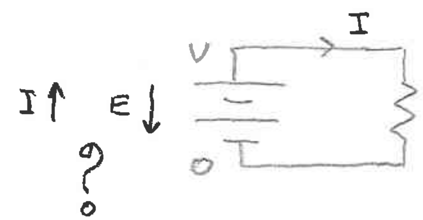
d) Next add a battery



$V = IR$ Although current flows, V changes only slowly, similar to a capacitor with huge capacitance. However, source of energy is chemical, not electrostatic.

- In a battery the charge moves in the direction opposite to the electric field
- Absurd since to excellent approximation only electrostatic forces are important.
- Need quantum mechanics!
- This magical souce of current, driving the internal current against the internal electric field is called an electromotive force or EMF.

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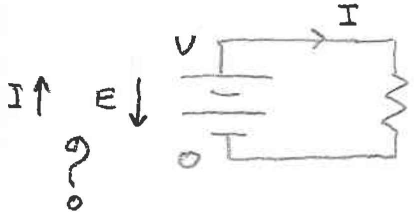

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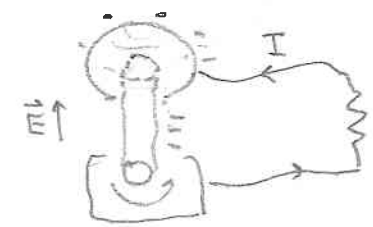
Actually there are two sources of EMF:

① $\oint_C \vec{E}\cdot d\vec{r} = 0$ and charge is forced by quantum mechanics to move against the $\vec{E}$ field

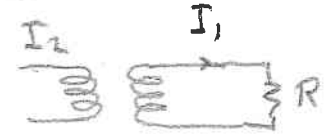
battery

or



Van de Graaff generator

② $\oint_C \vec{E}\cdot\vec{d} = -\frac{1}{c}\frac{d\Phi_B}{dt}$ Faraday's law



↑ large σ wire, $\vec{E} = 0$

$IR = EMF = M_{12}\frac{dI_2}{dt}$

mutual inductance

We can now solve a multitude of problems with resistors, capacitors and batteries connected in interesting ways