$$\frac{1}{c}\frac{d\Phi_B}{dt} = \overbrace{\frac{1}{c}\oint_S \frac{d\vec B}{dt}\cdot\hat n\, dS}^{\mathrm{I}} - \overbrace{\int_C \vec B\cdot(d\vec l\times\vec v)\frac{1}{c}}^{\mathrm{II}}$$

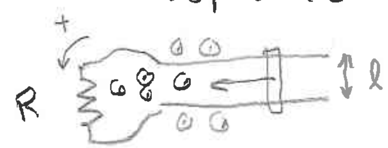
$$= -\int_S (\vec\nabla\times\vec E)\cdot\hat n\, dS - \int_C \left(\frac{\vec v}{c}\times\vec B\right)\cdot d\vec l$$

$$= -\int_C \underbrace{\left(\vec E + \frac{\vec v}{c}\times\vec B\right)}_{\vec E'}\cdot\hat n\, ds$$

$\vec E'$ in rest system of $C(t)$

5. Problem

top view

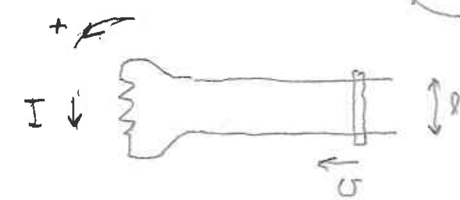


Bar of mass M slides down two conducting rails making an angle θ with the horizontal. There is a vertical magnetic field through which the conducting bar moves and a resistance R which completes the circuit. What is the bar's terminal velocity?

$$\oint \vec E\cdot d\vec l = -\frac{1}{c}\frac{d}{dt}\Phi_B(t) = +\frac{1}{c}\, l v B\cos\theta$$

$$I R = +\frac{1}{c}\, l v B\cos\theta$$

velocity of bar

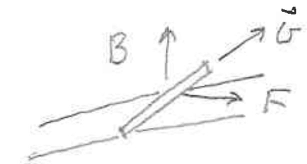


[I tends to increase Φ_B Lenz]

What is Lorentz force on current in wire? Let Q be total charge in wire flowing at velocity v

then $I = \frac{\Delta Q}{\Delta t} = \frac{Q}{l/v}$ or $vQ = lI$

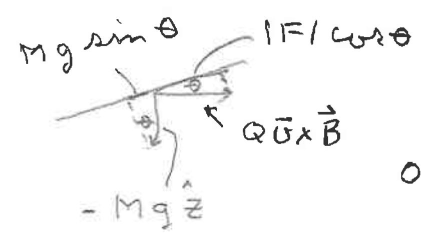
Look at bar



$\vec F = Q\frac{\vec v}{c}\times\vec B$

$\vec F$ is horizontal & $|F| = Qv\frac{B}{c} = lI\frac{B}{c}$

Balance $Mg\sin\theta = |F|\cos\theta$



$$Mg\sin\theta = \cos\theta\, l\,\frac{B}{c}\cdot\left[\frac{lvB}{Rc}\cos\theta\right]$$

or $$v = \frac{Mg\tan\theta\, Rc^2}{l^2B^2\cos\theta}$$

$$\oint \vec{E}\cdot d\vec{l} = -\frac{1}{c}\frac{d}{dt}\Phi_B(t) = +\frac{1}{c}\, l\, v\, B\cos\theta$$

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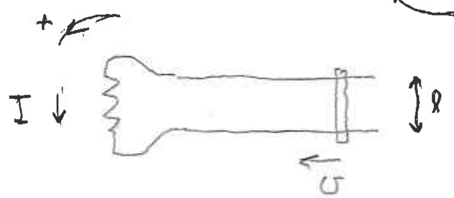

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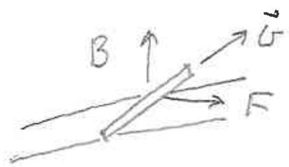

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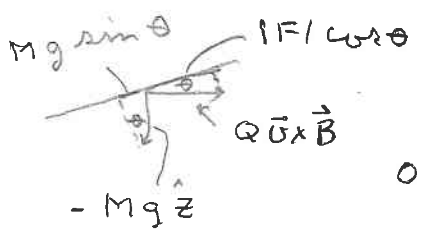

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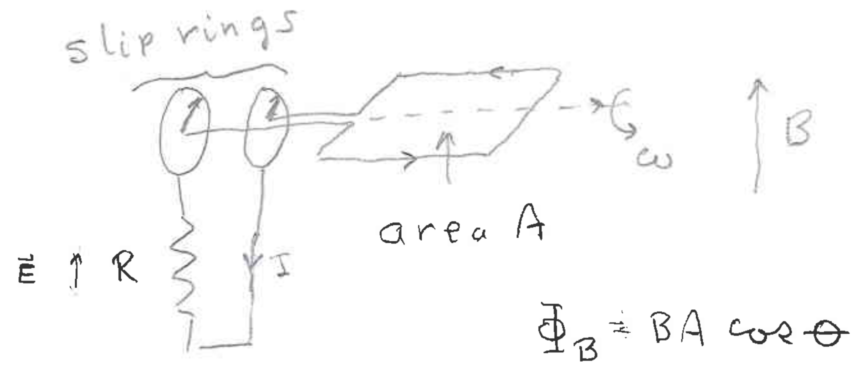
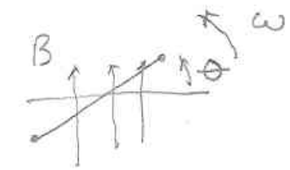
6. Simple AC generator



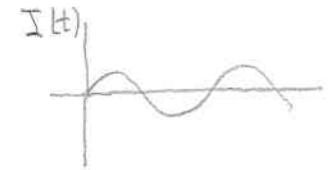
end view



$\Phi_B = BA\cos\theta$

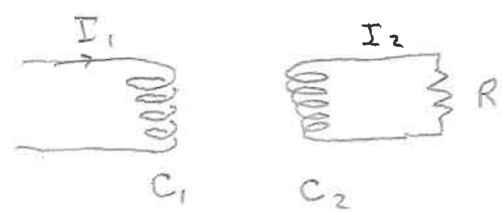
$$R I(t) = \oint_C \vec{E}\cdot d\vec{l} = -\frac{1}{c}\frac{d\Phi}{dt} = +\frac{1}{c} B A\omega\sin\omega t$$

$$I(t) = \frac{BA}{Rc}\,\omega\sin\omega t$$



Again I also creates B which tends to reverse change in Φ_B Lenz's law

7. Mutual inductance



changing current in C_1 will produce changing flux in C_2 & non-zero $\oint_{C_2} E\cdot dl \neq 0$

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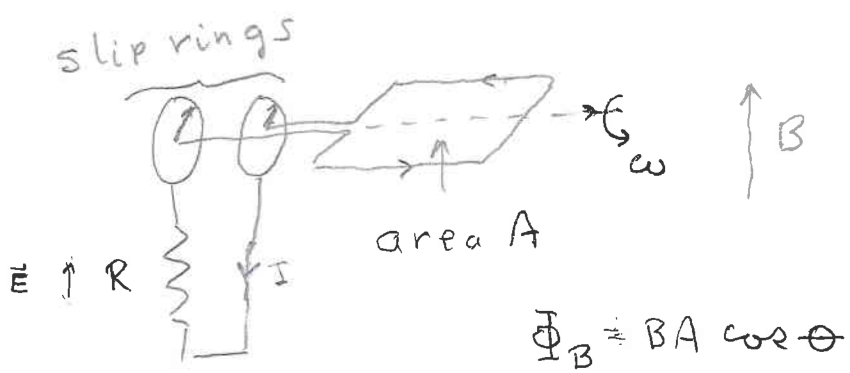
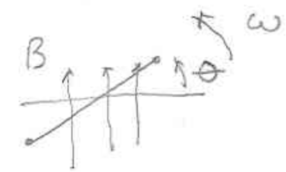


end view



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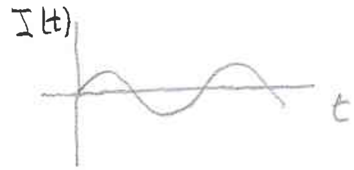

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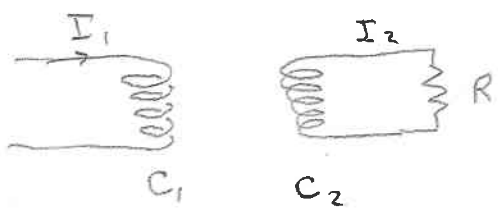

changing current in C_1 will produce changing flux in C_2 & non-zero

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The general formula for $\Phi_{2\text{ from }1}$ is revealing:

$$\Phi_{2\text{ from }1} = \int_{S_2} \vec{B}_1(\vec{r})\cdot\hat{n}\,dS$$

$$= \int_{S_2} [\vec{\nabla}\times\vec{A}_1(\vec{r})]\cdot\hat{n}\,dS$$

$$= \int_{C_2} \vec{A}_1(\vec{r}_2)\cdot d\vec{l}_2$$

$$= \int_{C_2} d\vec{l}_2\cdot\left[\int_{C_1}\frac{d\vec{l}_1}{|\vec{r}_1-\vec{r}_2|}\frac{I_1}{c}\right]$$

$$\int_{C_2}\vec{E}_1\cdot d\vec{l}_1 = -\frac{1}{c}\frac{d\Phi_{2\text{ from }1}}{dt} = -M_{21}\dot{I}_1$$

where the mutual inductance

$$M_{21} = \frac{1}{c^2}\int_{C_2} d\vec{l}_2\cdot\int_{C_1} d\vec{l}_1\,\frac{1}{|\vec{r}_1-\vec{r}_2|}$$

$M_{21} = M_{12}$ can thus be computed two ways: pick easier choice

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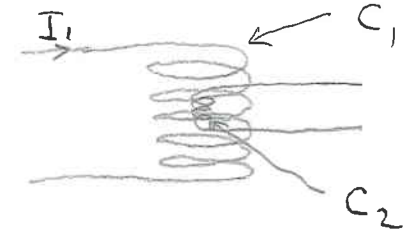
Example: A small coil C_2 has length l_2, radius r_2 and N_2 turns. A larger coil C_1 has length l_1, radius r_1, N_1 turns and contains C_1. Their symmetry axes coincide. Find M_{21}.



Field produced by I_1 inside C_1

$$\oint_{\partial S} \vec{B} \cdot d\vec{l} = \frac{4\pi}{c} \int_S \vec{J} \cdot \hat{n}\, dS$$

$$B_1 \Delta l = \frac{4\pi}{c} I \frac{N_1}{l_1} \Delta l$$

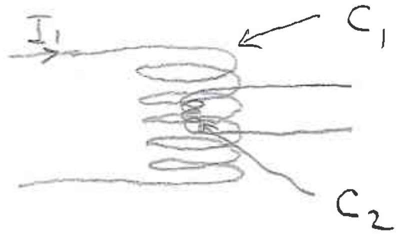
$$B_1 = \frac{4\pi I_1}{c} \frac{N_1}{l_1} \qquad \Phi_{2\text{ from }1} = B_1 \cdot \pi r_2^2 \cdot N_2$$

$$M_{21} = \frac{1}{c} \Phi_{2\text{ from }1} \frac{1}{I_1} = \frac{\pi r_2^2 N_2}{c} \frac{\frac{4\pi I_1 N_1}{c\, l_1}}{I_1}$$

$$= \frac{4\pi^2}{c^2} \frac{N_1 N_2}{l_1}$$

Note $M_{12} = M_{21}$ NOT obvious

Example: A small coil C_2 has length ℓ_2, radius r_2 and N_2 turns. A larger coil C_1 has length ℓ, radius r_1, N_1 turns and contains C_1. Their symmetry axes coincide. Find M_{21}.



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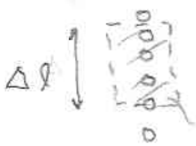

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$$B_1\,\Delta\ell = \frac{4\pi}{c} I \frac{N_1}{\ell_1}\Delta\ell$$

$$B_1 = \frac{4\pi I_1}{c}\frac{N_1}{\ell_1} \qquad \Phi_{2\ \text{from}\ 1} = B_1\cdot\pi r_2^2\cdot N_2$$

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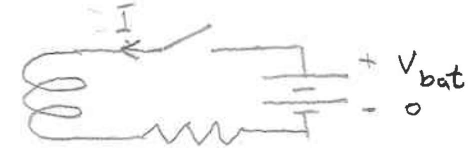
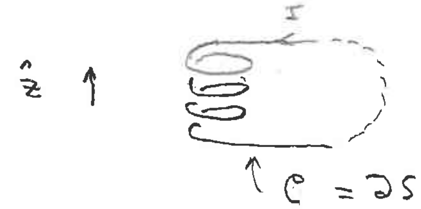
Self inductance:



Changing current will produce changing B which results in $\oint_C \vec{E}\cdot d\vec{\ell} \neq 0$!

For a coil with many turns most of the flux will be found in the coil



$$\Phi = \int_S \vec{B}\cdot\hat{n}\, ds$$

$$= \left[\frac{4\pi}{c}\cdot\frac{N}{\ell} I\right] N\pi r^2 I$$

$$\oint_C \vec{E}\cdot d\vec{\ell} = -\frac{1}{c}\frac{d\Phi}{dt} = -\underbrace{\frac{4\pi^2 N^2 r^2}{c^2 \ell}}_{L \,\equiv\, \text{inductance}}\cdot\frac{dI}{dt}$$

$$\therefore\ -L\frac{dI}{dt} = \oint_C \vec{E}\cdot d\vec{\ell}$$

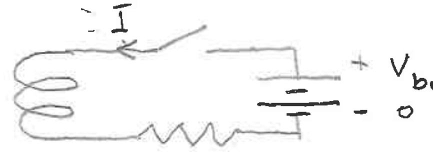
$$= IR - V_{bat} \quad \text{or} \quad V_{bat} = IR + L\frac{dI}{dt}$$

In general $L = \frac{1}{c^2}\oint_C d\vec{\ell}_1\cdot\oint_C d\vec{\ell}_2\,\frac{1}{|\vec{r}_1 - \vec{r}_2|}$

a) units

b) Find $I(t)$ after switch is closed

8. Self inductance:



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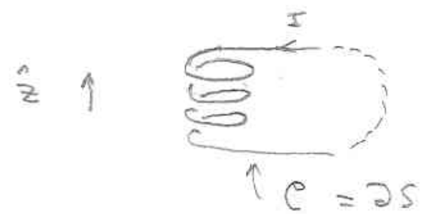

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- units
- Find $I(t)$ after switch is closed

9. units

esu: $\frac{sec^2}{cm}$

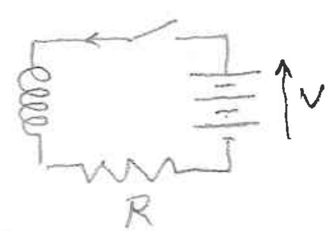
SI: $V = +L\frac{dI}{dt}$, $L \sim \frac{volts}{Coulomb}\, sec^2 \equiv$ Henry

$$\text{Henry} \sim \frac{\frac{1}{300}\,\text{statvolt}}{3\times10^9\,\text{esu}} \cdot sec^2 \quad (\text{statvolt} = \tfrac{esu}{cm})$$

$$\sim 1.1\times10^{-12}\ \frac{sec^2}{cm}$$

Since a simple coil with many turns $\sim 10^{-3}$ Henry, 1 Henry is large and one sec^2/cm is huge!

10.



$$V = IR + L\frac{dI}{dt}$$

$$\text{or } L\frac{d}{dt}I = -\underbrace{\left(I - \frac{V}{R}\right)}_{I'}R$$

$$\frac{d}{dt}I' = -I'\frac{R}{L}$$

$$I' = Ae^{-\frac{R}{L}t} \qquad I = I' + \frac{V}{R} = Ae^{-\frac{R}{L}t} + \frac{V}{R}$$

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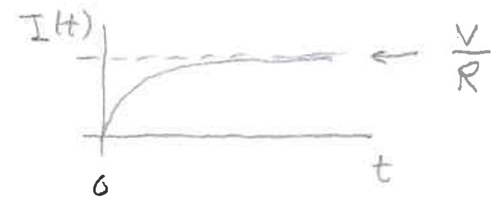
Choose the integration constant A so that $I(t) = 0$, when the switch closes at $t=0$: $A = -\dfrac{V}{R}$

$$I(t) = \frac{V}{R}\left(1 - e^{-\frac{R}{L}t}\right) \qquad R \sim \frac{sec}{cm} \qquad \frac{L}{R} \sim sec \checkmark$$

$$L \sim \frac{sec^2}{cm}$$

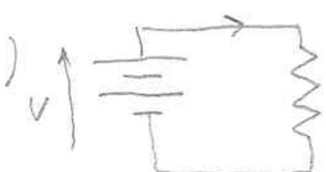


steady-state current is $I = \dfrac{V}{R}$. With $\dfrac{dI}{dt} = 0$, the inductor has no effect

---

11. Energy conservation: look at three examples:

a)

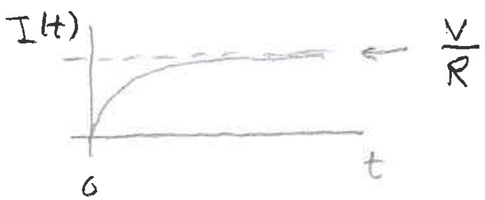


In time Δt, $\Delta Q = I\Delta t$ enters resistor at potential V & leaves resistor at potential 0. Energy lost in resistor is $\Delta U = V\Delta Q$

Choose the integration constant A so that $I(t) = 0$, when the switch closes at $t = 0$: $A = -\frac{V}{R}$

$$I(t) = \frac{V}{R}\left(1 - e^{-\frac{R}{L}t}\right) \qquad R \sim \frac{\text{sec}}{\text{cm}} \qquad \frac{L}{R} \sim \text{sec}$$

$$L \sim \frac{\text{sec}^2}{\text{cm}}$$



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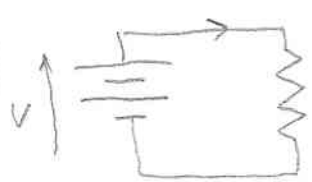


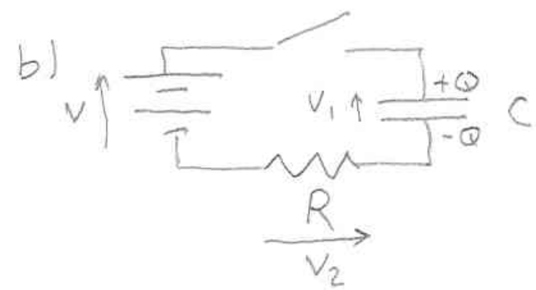
In time Δt, $\Delta Q = I \Delta t$ enters resistor at potential V & leaves resistor at potential 0. Energy lost in resistor is $\Delta U = V \Delta Q$

Power lost as heat in resistor

$$P = \frac{\Delta U}{\Delta t} = \frac{V \Delta Q}{\Delta t} = VI = \frac{V^2}{R} = I^2 R$$

---

b)



$$V = V_1 + V_2$$

$$V_2 = IR \qquad V_1 = \frac{Q}{C}$$

$$\frac{dQ}{dt} = I$$

power provided by battery $P_{bat} = VI$

power dissapated in resistor $P_R = V_2 I$

energy stored in capacitor $U_C = \frac{1}{2}\frac{Q^2}{C}$

power needed to charge capacitor

$$P_C = \frac{d}{dt}U_C = \frac{1}{2}\frac{d}{dt}\frac{Q^2}{C} = \frac{Q}{C} \cdot I = V_1 I$$

$$P_{bat} = I(V_1 + V_2) = P_C + P_R$$

& energy is conserved.

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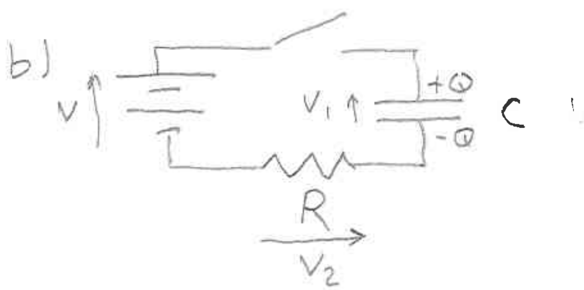

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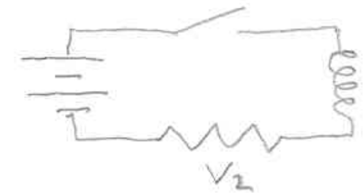
c)



$$P_{bat} = IV$$

$$R_R = IV_2$$

Power absorbed by inductor

$$P_L = P_{bat} - P_R = I(V - V_2) = IL\frac{dI}{dt}$$

Thus energy stored in inductor obeys $\frac{dU_L}{dt} = IL\frac{dI}{dt}$ or $U_L = \frac{1}{2}LI^2$

consider a coil of radius r, length l and N turns

$$L = \frac{4\pi^2 N^2 r^2}{c^2 l} \qquad B = \frac{4\pi}{c}\cdot\frac{N}{l}\cdot I$$

$$U_L = \frac{1}{2}\frac{4\pi^2 N^2 r^2}{c^2 l}\cdot\left[\frac{c}{4\pi}\frac{l}{N}B\right]^2$$

$$= \frac{1}{8} r^2 l B^2 = \frac{1}{8\pi}\underbrace{(\pi r^2 l)}_{\text{volume in solenoid}} B^2$$

Thus E&M energy densitys $= \dfrac{\vec{E}^2 + \vec{B}^2}{8\pi}$

c)

$$P_{bat} = IV$$

$$R_R = I V_2$$

Power absorbed by inductor

$$P_L = P_{bat} - P_R = I(V - V_2) = I L \frac{dI}{dt}$$

Thus energy stored in inductor obeys

$$\frac{dU_L}{dt} = I L \frac{dI}{dt} \quad \text{or} \quad U_L = \frac{1}{2} L I^2$$

consider a coil of radius r, length ℓ and N turns

$$L = \frac{4\pi^2 N^2 r^2}{c^2 \ell} \qquad B = \frac{4\pi}{c} \cdot \frac{N}{\ell} \cdot I$$

$$U_L = \frac{1}{2} \frac{4\pi^2 N^2 r^2}{c^2 \ell} \cdot \left[\frac{c}{4\pi} \frac{\ell}{N} B\right]^2$$

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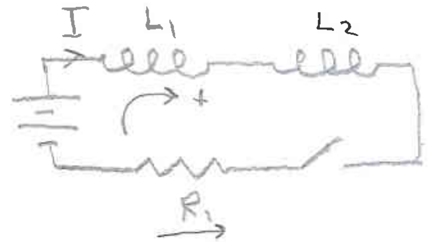
12. Combine inductors



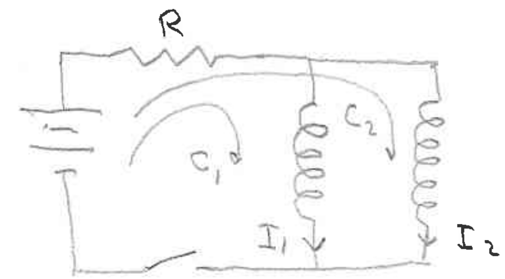
$$\oint \vec{E} \cdot dt = -\frac{1}{c} \frac{d\Phi}{dt}$$

$$= -\frac{1}{c} \frac{d\Phi_1}{dt} - \frac{1}{c} \frac{d\Phi_2}{dt}$$

$$-V + IR = -L_1 \frac{dI}{dt} - L_2 \frac{dI}{dt}$$

$$\text{or} \quad V = IR + \underbrace{(L_1 + L_2)}_{L} \frac{dI}{dt}$$

Inductors in series add.



$$I = I_1 + I_2$$

$$\oint_{C_1} \bar{E} \cdot dr = -L_1 \frac{dI_1}{dt}$$

$$-V + RI = -L_1 \frac{dI_1}{dt} \qquad -V + RI = -L_2 \frac{dI_2}{dt}$$

divide by L_1 divide by L_2

add:

$$(-V + RI)\left(\frac{1}{L_1} + \frac{1}{L_2}\right) = -\frac{dI_1}{dt} - \frac{dI_2}{dt} = -\frac{dI}{dt}$$

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Inductors in parallel combine like resistors in parallel

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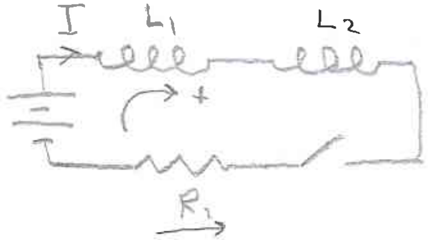

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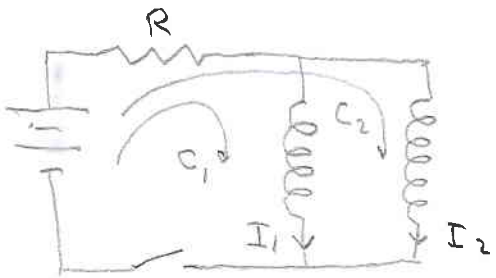

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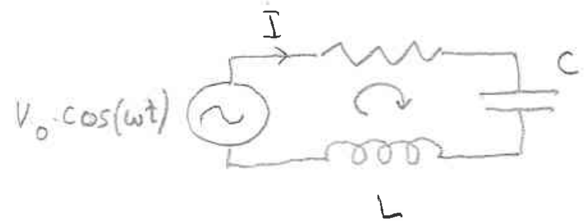
D. AC circuits

1. Consider circuits driven by a simple oscillating voltage $V(t) = V_0 \cos\omega t$. Could derive from Con Ed an incident radio wave, ...



$$\oint \vec{E}\cdot d\vec{l} = -L\frac{dI}{dt}$$

$$-V_0\cos\omega t + IR + \frac{Q}{C} = -L\frac{dI}{dt}$$

or $$L\frac{dI}{dt} + RI + \frac{Q}{C} = V_0\cos\omega t$$

or $$L\frac{d^2Q}{dt^2} + R\frac{dQ}{dt} + \frac{Q}{C} = V_0\cos\omega t$$

the same equation as forced S.H.M.

$$m\frac{d^2x}{dt^2} + \sigma\frac{dx}{dt} + kx = F_0\cos\omega t$$

easily solve by introducing a second equation

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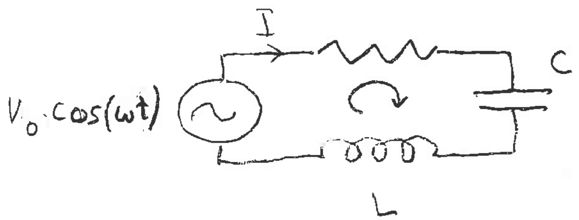

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Calling the solution to the first equation Q_r, multiplying the second equation by "i" & adding

$$L\frac{d^2}{dt^2}\underbrace{(Q_r + iQ_i)}_{Q} + R\frac{d}{dt}(Q_r + iQ_i) + \frac{1}{C}(Q_r + iQ_i) = V_0\underbrace{(\cos\omega t + i\sin\omega t)}_{e^{i\omega t}}$$

Thus, solve

$$L\frac{d^2}{dt^2}Q + R\frac{d}{dt}Q + \frac{1}{C}Q = V_0 e^{i\omega t}$$

and find the real part of Q