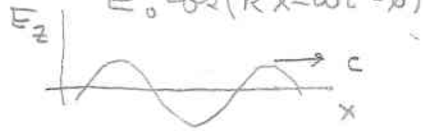
Two important facts:

(A) E_z $E_0 \cos(kx - \omega t - \phi)$

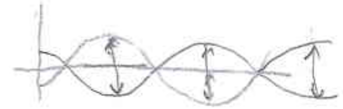


+

E_z

$E_0 \cos(kx + \omega t + \phi)$

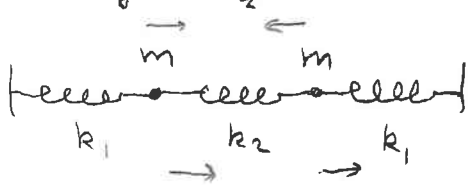
= $E_0 \cos kx \cos(\omega t + \phi)$



fixed pattern in space, amplitude oscillates

(B) Wave equation $\frac{d^2 E_z}{dx^2} = + c^2 \frac{d^2}{dx^2} E_z$

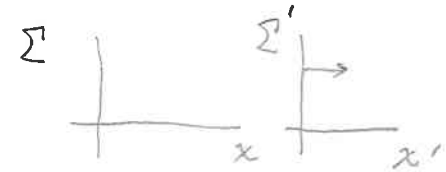
becomes a SHO problem if we choose $E_z(x,t) = E_z(t) \cos(kx)$ for any k! The wave equation contains an infinite number of SHO problems. These coupled SHO behaviors, each with different frequencies can be separated by using $E_z(t)\cos(kx)$! Similar to



two modes each with a different frequency

---

4. Examine how our traveling wave solution Lorentz transforms



$x' = \gamma(x - vt)$

$t' = \gamma(t - \frac{v}{c^2} x)$

assume in Σ' $E'_z(x',t') = E'_0 \cos(k'x' - \omega' t')$

$B'_y(x',t') = -E'_0 \cos(k'x' - \omega' t')$

find $\vec{E}(x,t)$ + $\vec{B}(x,t)$ in Σ system

$E_\parallel = E'_\parallel = 0 \qquad B_\parallel = B'_\parallel = 0$

$E_\perp = \gamma(E'_\perp - \frac{\vec{v}}{c} \times B')$

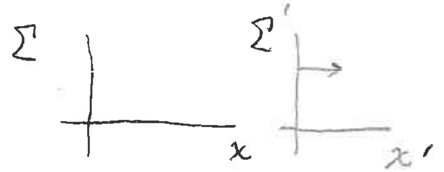
$E_z(x,t) = \gamma(1 + \frac{v}{c}) E'_0 \cos[k'x' - \omega' t']$

$k'\gamma(x - vt) - \omega'\gamma(t - \frac{v}{c^2}x)$

$= x \underbrace{\gamma(k' + \frac{v}{c^2}\omega')}_{k} - t \underbrace{\gamma(\omega' + vk')}_{\omega}$

$= \underbrace{\sqrt{\frac{1 + \frac{v}{c}}{1 - v/c}}\, E'_0}_{E_0} \cos(kx - \omega t)$

4. Examine how our traveling wave solution Lorentz transforms



$$x' = \gamma(x - vt)$$
$$t' = \gamma\left(t - \frac{v}{c^2}x\right)$$

assume in Σ' $E'_z(x',t') = E'_0 \cos(k'x' - \omega' t')$

$$B'_y(x',t') = -E'_0 \cos(k'x' - \omega' t')$$

find $\vec{E}(x,t)$ + $\vec{B}(x,t)$ in Σ system

$$E_\parallel = E'_\parallel = 0 \qquad B_\parallel = B'_\parallel = 0$$

$$E_\perp = \gamma\left(E'_\perp - \frac{\vec{v}}{c}\times B'\right)$$

$$E_z(x,t) = \gamma\left(1+\frac{v}{c}\right)E'_0 \cos\left[k'x' - \omega' t'\right]$$

$$k'\gamma(x - vt) - \omega'\gamma\left(t - \frac{v}{c^2}x\right)$$
$$= x\,\underbrace{\gamma\left(k' + \frac{v}{c^2}\omega'\right)}_{k} - t\,\underbrace{\gamma\left(\omega' + vk'\right)}_{\omega}$$

$$= \underbrace{\sqrt{\frac{1+\frac{v}{c}}{1-v/c}}\,E'_0}_{E_0}\cos(kx - \omega t)$$

note

$$k = \gamma\left(k' + \frac{v}{c^2}\omega'\right) = \gamma\left(k' + \frac{v}{c}\frac{\omega'}{c}\right)$$
$$\frac{\omega}{c} = \frac{1}{c}\gamma(\omega' + vk') = \gamma\left(\frac{\omega'}{c} + \frac{v}{c}k'\right)$$

so $\left(k, 0, 0, \frac{\omega}{c}\right)$ transforms like a four-vector

also $\omega = \gamma(\omega' + v\overset{\omega'/c}{k'}) = \gamma\omega'\left(1 + \frac{v}{c}\right)$

$= \omega'\left[\frac{1+\frac{v}{c}}{1-v/c}\right]^{1/2}$ our old Doppler effect formula

Recall for a massless particle

$$p = \sqrt{E^2/c^2 - m^2c^2} \rightarrow E/c$$

for a photon $E = h\nu = \frac{h}{2\pi}\cdot 2\pi\nu = \hbar\omega$

$$p = \frac{E}{c} = \frac{\hbar\omega}{c} = \hbar k$$

so $\left(p, 0, 0, \frac{E}{c}\right) = \hbar(k, 0, 0, \omega)$

is a relation between two four-vectors made of a photon's momentum and energy and it wave-number & frequency note

$$k = \gamma\left(k' + \frac{v}{c^2}\omega'\right) = \gamma\left(k' + \frac{v}{c}\frac{\omega'}{c}\right)$$

$$\frac{\omega}{c} = \frac{1}{c}\gamma(\omega' + vk') = \gamma\left(\frac{\omega'}{c} + \frac{v}{c}k'\right)$$

so $(k, 0, 0, \frac{\omega}{c})$ transforms like a four-vector

also $\omega = \gamma(\omega' + v\overset{\omega'/c}{k'}) = \gamma\omega'(1 + \frac{v}{c})$

$= \omega'\left[\frac{1+\frac{v}{c}}{1-\frac{v}{c}}\right]^{1/2}$ our old Doppler effect formula

Recall for a massless particle

$p = \sqrt{E^2/c^2 - m^2c^2} \to E/c$

for a photon $E = h\nu = \frac{h}{2\pi}\cdot 2\pi\nu = \hbar\omega$

$$p = \frac{E}{c} = \frac{\hbar\omega}{c} = \hbar k$$

so $(p, 0, 0, \frac{E}{c}) = \hbar(k, 0, 0, \omega)$

is a relation between two four-vectors made of a photon's momentum and energy and it wave-number & frequency

VI Quantum Mechanics

A Overview

1. Newtonian view of reality fails when applied to phenomena occuring at small energies E and short times:

$$E \cdot t \sim \hbar = 1.06 \times 10^{-27} \text{ erg}\cdot\text{sec}$$

$\hbar = \frac{h}{2\pi}$ h Plank's constant

Note $\hbar$ has the same units as distance x momentum and as angular momentum. Newton's view fails when

$$x \cdot p \approx \hbar \quad \text{or} \quad L \approx \hbar$$

Plank's constant sets the scale for atomic phenomena:

VI Quantum Mechanics

A Overview

1. Newtonian view of reality fails when applied to phenomena occuring at small energies E and short times:

$$E \cdot t \sim \hbar = 1.06 \times 10^{-27} \text{ erg} \cdot \text{sec}$$

$\hbar = \frac{h}{2\pi}$ ← Plank's constant

Note $\hbar$ has the same units as distance x momentum and as angular momentum, Newton's view fails when

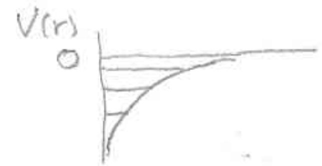
$$x \cdot p \approx \hbar \quad \text{or} \quad L \approx \hbar$$

Plank's constant sets the scale for atomic phenomena:

a) Atoms emit light a special frequencies which are neither continuous nor harmonic multiples of a fundamental

$$\nu \sim \frac{1}{n_2^2} - \frac{1}{n_1^2}$$

Atoms are miniature planetary systems with discrete energies



Stablized againts losing energy to radiation because they have a lowest energy.

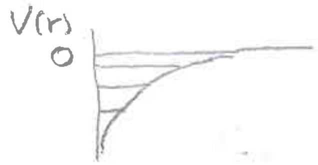
$$E_n \propto \frac{1}{n^2}$$

emitted frequencies $\nu \sim \frac{E_{n_2}}{\hbar} - \frac{E_{n_1}}{\hbar}$

a) Atoms emit light a special frequencies which are neither continuous nor harmonic multiples of a fundamental

$$\nu \propto \frac{1}{n_2^2} - \frac{1}{n_1^2}$$

Atoms are miniature planetary systems with discrete energies

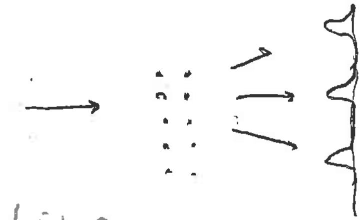


Stablized againts losing energy to radiation because they have a lowest energy.

$$E_n \propto \frac{1}{n^2}$$

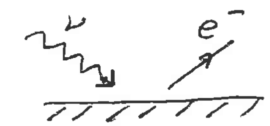
emitted frequencies $\nu \sim \frac{E_{n_2}}{\hbar} - \frac{E_{n_1}}{\hbar}$

b) Monoenergetic electrons scatter from a crystal at fixed angles just like light scattering from a diffraction grating



$$\lambda \sim \frac{h}{p}$$

c) Light incident on the surface of a metal can knock out electrons only when $\nu > \nu_{min}$: no emission if $\nu < \nu_{min}$ even at high intensity

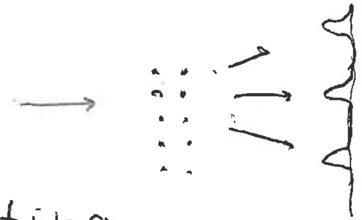


Results from light of a single frequency ν being made of many "photons" each with energy $h\nu$.

i) Need $h\nu + E^{e^-}_{\text{ground state}} > 0$ to kick e^- free

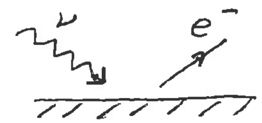
ii) Unlikely for two photons to hit the same electron at the same time.

b) Monoenergetic electrons scatter from a crystal at fixed angles just like light scattering from a diffraction grating



$$\lambda \sim \frac{h}{p}$$

c) Light incident on the surface of a metal can knock out electrons only when $\nu > \nu_{min}$: no emission if $\nu < \nu_{min}$ even at high intensity



Results from light of a single frequency ν being made of many "photons" each with energy $h\nu$.

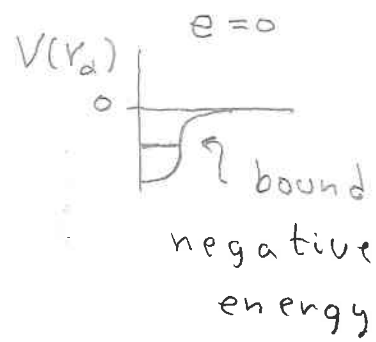
i) Need $h\nu + E^{e^-}_{\text{ground state}} > 0$ to kick e^- free

ii) Unlikely for two photons to hit the same electron at the same time.

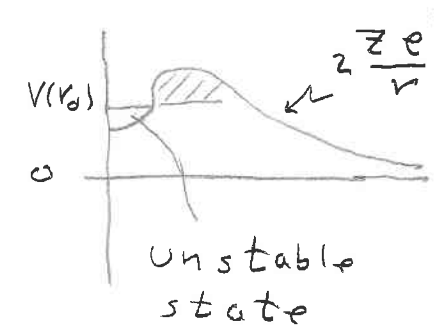
d) Radioactive nuclei at a constant rate with a lifetime (the inverse of the decay rate) which can be very long ~

$$U^{92}_{238} \rightarrow Th^{90}_{234} + \alpha \qquad \tau = 4.5\times10^{9}\text{ years}$$

How could you construct a classical system that would behave this way?



include Coulomb repulsion $\longrightarrow$

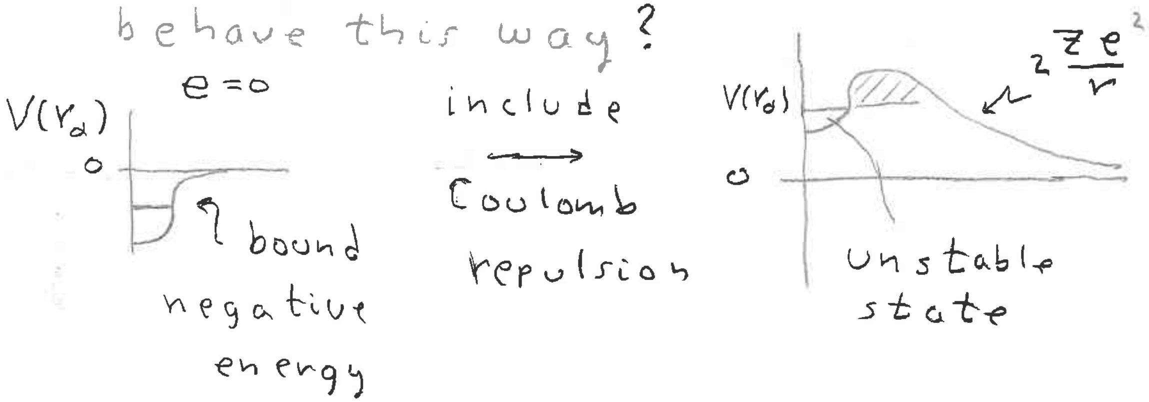


Now α particle has positive energy and can "tunnel" through barrier and escape!

d) Radioactive nuclei at a constant rate with a lifetime (the inverse of the decay rate) which can be very long ~

$$U^{92}_{238} \rightarrow Th^{90}_{234} + \alpha \qquad \tau = 4.5\times10^{9} \text{ years}$$

How could you construct a classical system that would behave this way?



Now a particle has positive energy and can "tunnel" through barrier and escape!

B Quantum Mechanics of angular momentum $\vec{J}$

strategy:
- study a simpler problem than usual wave functions
- postulate known QM theory
- focus on mathematical foundation

1. Begin by discussing only J_z; adding J_x and J_y is highly non-trivial and reveals most of quantum physics

Classically, if $\vec{J}^2$ is known, then J_z can vary continouously between $\pm\sqrt{\vec{J}^2}$: $\quad -\sqrt{\vec{J}^2} \leq J_z \leq +\sqrt{\vec{J}^2}$

B Quantum Mechanics of angular momentum $\vec{J}$

strategy:
- study a simpler problem than usual wave functions
- postulate known QM theory
- focus on mathematical foundation

1. Begin by discussing only J_z; adding J_x and J_y is highly non-trivial and reveals most of quantum physics

Classically, if $\vec{J}^2$ is known, then J_z can vary continously between $\pm\sqrt{\vec{J}^2}$: $-\sqrt{\vec{J}^2} \le J_z \le +\sqrt{\vec{J}^2}$

In quantum mechanics for a system with a specific $\vec{J}^2$, J_z takes discrete values $J_z = m\hbar$ where m is an integer or an integer $+\frac{1}{2}$ and $-j \le m \le j$ where j also is an integer or half integer:

- simplest case $j = \frac{1}{2}$: $m = +\frac{1}{2}$ or $m = -\frac{1}{2}$
- next case $j = 1$: $m = 1, 0, -1$
- next case $j = \frac{3}{2}$ $m = -\frac{3}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{3}{2}$

number of allowed values is $2j+1$

In classical mechanics a point particle is represented by two vectors, its position and momentum $\vec{r}, \vec{p}$. $\vec{r}$ & $\vec{p}$ tell us everything we need to know and allow motion prediction.

In quantum mechanics for a system with a specific $\vec{J}^2$, J_z takes discrete values $J_z = m\hbar$ where m is an integer or an integer $+\frac{1}{2}$ and $-j \le m \le j$ where j also is an integer or half integer:

- simplest case $j = \frac{1}{2}$: $m = +\frac{1}{2}$ or $m = -\frac{1}{2}$
- next case $j = 1$: $m = 1, 0, -1$
- next case $j = \frac{3}{2}$ $m = -\frac{3}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{3}{2}$

number of allowed values is $2j+1$

In classical mechanics a point particle ~~is represented~~ by two vectors, its position and momentum $\vec{r}, \vec{p}$.

$\vec{r}$ & $\vec{p}$ tell us everything we need to know and allow motion prediction.

How do we represent our quantum mechanical particle with angular momenty $J_z = \hbar m$ $-j \le m \le j$?

Each value of m corresponds to an orthonormal vector $|m\rangle$. Thus we need a $2j+1$ dimensional vector space to describe a system with angular momentum of magnitude j.

If our system has $J_z = \hbar m$ it is represented by the state $|m\rangle$.

In quantum mechanic we give meaning to the general state

$$|\psi\rangle = \sum_{m=-j}^{j} \psi_m |m\rangle$$

complex numbers!

How do we represent our quantum mechanical particle with angular momenty $J_z = \hbar m$ $-j \le m \le j$?

Each value of m corresponds to an orthonormal vector $|m\rangle$, Thus we need a $2j+1$ dimensional vector space to describe a system with angular momentum of magnitude j.

If our system has $J_z = \hbar m$ it is represented by the state $|m\rangle$.

In quantum mechanic we give meaning to the general state

$$|\psi\rangle = \sum_{m=-j}^{j} \psi_m |m\rangle$$

complex numbers !

Thus, we work with a complex vector space where we can multiply vectors by complex numbers

What does $|\psi\rangle = \sum_{m=-j}^{j} \psi_m |m\rangle$ mean ?

This state does not have a definite value of J_z. Instead if J_z is measured we will obtain the value $\hbar m$ with probability $|\psi_m|^2$.

Very different from the 6-D vector space of $\vec{r}$ and $\vec{p}$:

$\vec{r}_1 + \vec{r}_2$, $\vec{p}_1 + \vec{p}_2$ describes a new situation with a particle at a new position $\vec{r}_1 + \vec{r}_2$ with momentum $\vec{p}_1 + \vec{p}_2$