Physics UN2802 | Professor Christ
due 03/31/2022

Assignment #19
Definition of notation added to problem 164.

Reading:

March 29 French and Taylor chapter 3-1 thru 3-6
March 31 French and Taylor chapter 3-6 thru 3-11

Problems:

157. Purcell 8.16

158. Purcell 9.2

159. Purcell 9.7

160. Purcell 9.9

161. Purcell 9.12

162. French and Taylor 1-11

163. French and Taylor 1-12

164. Show by explicit matrix multiplication that the following 3×3 matrices:

$$J_1 = \frac{\hbar}{\sqrt{2}} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \quad J_2 = \frac{\hbar}{\sqrt{2}} \begin{pmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{pmatrix} \quad J_3 = \hbar \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}.$$

obey the commutation relations expected of angular momentum operators:

$$[J_i, J_j] = i\hbar \sum_{k=1}^{3} \epsilon_{ijk} J_k.$$

To what value of j do they correspond?

Definition of notation:

- If O_1 and O_2 are two linear operators, the commutator of O_1 and O_2 is defined as $[O_1, O_2] = O_1 O_2 - O_2 O_1$.
- The Levi-Civita density ϵ_{ijk} is a three-index tensor which changes sign when any pair of its three indices are exchanged. For example, $\epsilon_{123} = -\epsilon_{321}$. Knowing that $\epsilon_{123} = 1$ allows us to determine ϵ_{Ijk} for all values of i, j and k.

165. Show from the basic commutation relations between J_i and J_j that $[J_x, \vec{J}^{\,2}] = 0$ where $\vec{J}^{\,2} = J_1^2 + J_2^2 + J_3^2$.