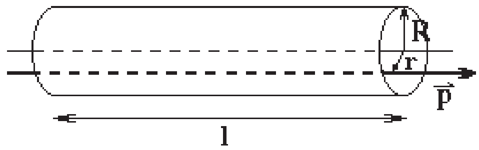
Physics UN2802 | Professor Christ
Final Examination (practice) | 05/05/2022

Answer three of the following **five (5)** questions. Please give a complete description of your method of solution since *partial credit* will be given.

1. A cylinder of length l and radius R carries a uniform current density j.

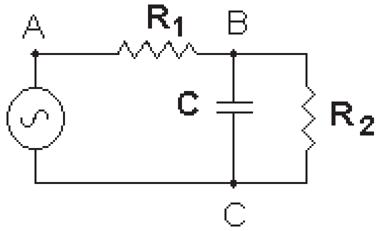


   (a) Find the magnetic field for both the regions $r > R$ and $r < R$. [13 points]

   (b) A particle with momentum $\vec{p}$ and charge q moves parallel to and within the cylinder, displaced a distance r from the cylinder's axis. Calculate the additional momentum $\vec{\Delta p}$ given the particle as it passes through the length l of the cylinder. (You should assume that $\Delta p/p \ll 1$ so that the particle's spatial deflection by the magnetic forces can be neglected.) [10 points]

   (c) Including the effects of this $\vec{\Delta p}$ on the subsequent motion of the particle, show that such particles for all values of r, $0 \le r \le R$, will cross the axis of the cylinder at the same distance D from the cylinder. Find this focal length D. [10 points]

2. Consider the combination of two resistors, R_1 and R_2 and a capacitor C driven by an applied voltage $V(t) = V_0 \cos(\omega t)$ configured as shown at the right.

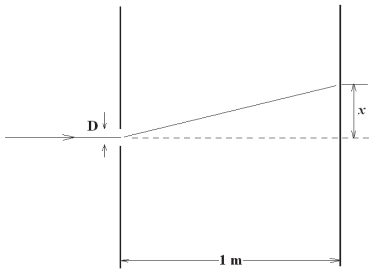


   (a) Find the complex impedance Z seen between the points A and C. [11 points]

   (b) What is the actual current flowing through R_1 in both amplitude and phase? [11 points]

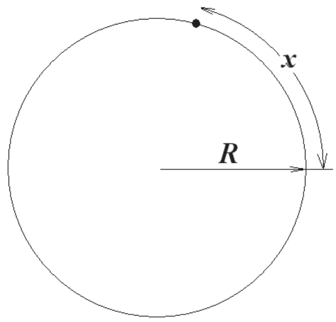
   (c) What is the average power dissipated in the resistor R_2? [11 points]

3. (a) A mono-energetic, horizontal beam of electrons, each with an energy of 1 electron Volt, is incident on a vertical screen with a horizontal slit of width $D = 10^{-5}$ cm. Make a rough numerical estimate of the vertical width of the pattern of electrons that strike a vertical screen a distance of 1 meter from the screen. Recall that an electron has mass $m = 0.911\ 10^{-27}$ gram, $\hbar = 1.05\ 10^{-27}$ erg sec and 1 eV $= 1.602\ 10^{-12}$ erg. [11 points]



(b) Use the uncertainty principle, $\Delta r\ \Delta p \geq \hbar/2$ to show that the orbit of an electron bound to a proton must have a minimum size, recognizing that the orbiting electron will have energy equal to $p^2/2m - e^2/r$. Estimate the size of the hydrogen atom in terms of the electron's mass (m), charge (e) and Planck's constant $\hbar$ [11 points]

(c) A particle of mass m and charge q is constrained to move in a circle of radius R. The particle is located by giving its position x measured along the circumference. If a uniform magnetic field B is imposed perpendicular to the plane of the circle, the energy operator can be written:



$$H_{\text{op}} = \frac{(p_{\text{op}} - eRB/c)^2}{2m}.$$

Find the allowed energies and the wave functions of the corresponding energy eigenstates for this system. [11 points]

4. A particle of mass m moves within a very deep square well $V(x)$ given by

$$V(x) = \begin{cases} V_0 & x \leq 0 \text{ or } L \leq x \\ 0 & 0 \leq x \leq L \end{cases} \tag{1}$$

where V_0 is a very large positive constant.

(a) Write down the energy operator (Hamiltonian) for this system and determine those allowed energies which are much smaller than V_0 and the corresponding eigenstates. [9 points]

(b) Consider a state which at $t = 0$ has the wave function

$$\psi(x, 0) = N \sin(\frac{2\pi}{L}x) \cos(\frac{\pi}{L}x). \tag{2}$$

Find the normalization constant N needed so that the state $\psi(x, 0)$ has unit probability. [5 points]

(c) If the energy of this state is measured, what results are possible? With what probabilities will each of those results be found? [6 points]

(d) Find the state $\psi(x, t)$ into which the state above evolves at the time t. [8 points]

(e) Determine the smallest non-zero value of t for which $\psi(x, t)$ is physically equivalent to $\psi(x, 0)$. [5 points]

5. Consider the quantum mechanics of a spin-1/2 particle with angular momentum $\vec{J}$ and magnetic moment $\vec{\mu} = \gamma \vec{J}$. Introduce two normalized eigenstates of J_z, $|+\frac{1}{2}\rangle_z$ and $|-\frac{1}{2}\rangle_z$ with phases chosen so that when this basis is used the three angular momentum operators J_x, J_y and J_z become $\hbar/2$ times the standard Pauli matrices:

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{3}$$

Initially the system is in the state

$$|\psi\rangle = \frac{1}{3}\left|+\frac{1}{2}\right\rangle_z - \frac{2\sqrt{2}}{3}\left|-\frac{1}{2}\right\rangle_z \tag{4}$$

(a) What is the probability that a measurement of J_z will return the value $+\hbar/2$? The value $-\hbar/2$? or the value 0? [6 points]

(b) If the value $-\hbar/2$ is found for J_z, what will be the resulting state? [2 points]

(c) If this same state given in Eq. (4) is prepared many times and the angular momentum in the x-direction measured, what will be the average result of these measurements? If instead J_y is measured what will be the result of a similar average of many measurements? [10 points]

(d) Next at $t = 0$ a magnetic field of magnitude B pointing in the $+\hat{z}$ direction is applied to the state $|\psi\rangle$ above. What will be the resulting state $|\psi(t)\rangle$ after a time t has elapsed? [8 points]

(e) As in part (c) consider the case where this same state $|\psi(t)\rangle$ is prepared many times and J_x measured. What will be the average result of these measurements? What will be the average result if instead J_y is measured? [7 points]