1/18/2022 ①

UN 2802 - 1st four lectures on line

- Complete E & M using Purcell 2nd edition
- Introduce quantum mechanics using French & Taylor and my notes

course website

www. columbia. edu/~nhc1/UN2802

organizational details like last semester

- Problem sets due on Thursdays
- Grade obtained from 30% midterm, 20% homework 50% final
- Some Python calculations in class and some Python problems in problem sets

problem session instructor and graders to be determined along with meeting time and room

my email address

nhc@phys.columbia.edu

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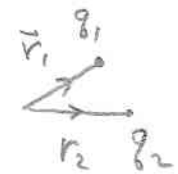
We have covered most of electrostatics in UN2801

- Coulomb's law



$$\vec{F}_{1 \text{ on } 2} = q_1 q_2 \frac{\vec{r}_2 - \vec{r}_1}{|\vec{r}_2 - \vec{r}_1|^3}$$

- Electric field

$$\vec{E}(\vec{r}) = \sum_{i=1}^{N} q_i \frac{\vec{r} - \vec{r}_i}{|\vec{r} - \vec{r}_i|^3}$$

q_i

$$\vec{F}_{\substack{\text{on } \delta q \\ \text{at } \vec{r}}} = \delta q \, \vec{E}(\vec{r})$$

- Gauss' law

$$\int_{\partial V} \vec{E}(\vec{r}) \cdot \hat{n} \, ds = 4\pi \int_V \rho(r) \, dV$$

- Gauss' theorem

$$\int_{\partial V} \vec{E}(\vec{r}) \cdot \hat{n} \, ds = \int_V \vec{\nabla} \cdot \vec{E} \, dV$$

$$\Rightarrow \vec{\nabla} \cdot \vec{E}(\vec{r}) = 4\pi \rho(\vec{r})$$

- Scalar potential

$\vec{\nabla} \times \vec{E} = 0 \Rightarrow \vec{E} = -\vec{\nabla} \phi$

energy of δq at $\vec{r}$ is $\delta q \, \phi(\vec{r})$

$\vec{\nabla} \cdot \vec{E} = 4\pi \rho \Rightarrow \vec{\nabla}^2 \phi = -4\pi \rho$ Poisson equation

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Repeat final topic of 2021:

Deepen our understanding of the electric field by expressing the potential energy of a charge distribution $\rho(\vec{r})$ in terms of the electric field it produces

$$U = \frac{1}{2} \int_V d^3\vec{r}_1 \int_V d^3 r_2 \, \rho(\vec{r}_1) \underbrace{\frac{1}{|\vec{r}_1 - \vec{r}_2|}} \rho(\vec{r}_2)$$

$$= \frac{1}{2} \int_V d^3 r_1 \underbrace{\rho(r_1)}_{\frac{1}{4\pi} \vec{\nabla} \cdot \vec{E}(\vec{r}_1)} \underbrace{\int_V d^3 r_2 \frac{1}{|\vec{r}_1 - \vec{r}_2|} \rho(\vec{r}_2)}_{\Phi(\vec{r}_1)}$$

$$= \frac{1}{8\pi} \int_V d^3 r_1 \, \vec{\nabla} \cdot \left(\vec{E}(\vec{r}_1) \Phi(\vec{r}_1) \right) - \frac{1}{8\pi} \int_V d^3 r_1 \left(\vec{E}(\vec{r}_1) \cdot \underbrace{\vec{\nabla} \Phi(\vec{r}_1)}_{-\vec{E}(\vec{r}_1)} \right)$$

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$\rightarrow 0$ for large R

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E Conductors

1. Electrical properties of materials

Recall material are made of atoms with a heavy compact positive charge at their center surrounded by a much larger cloud of negatively charged electrons. Two common behaviors are seen when an electric field $\vec{E}$ is applied.

insulator a) Electrons stay bound to a nucleus or cluster of nuclei and $\vec{E}(\vec{r}) \neq 0$ does not make the travel.

conductor b) One or more electrons associated with some types of atom can move from one atom the the next and travel a macroscopic distance when $\vec{E}$ is applied, moving in the direction of $-\vec{E}(\vec{r})$.

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2. Simple theory of conduction:

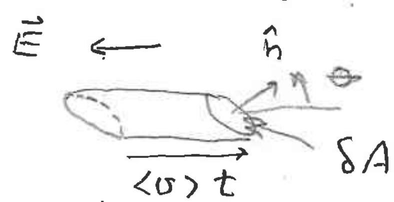
$$\vec{a}_e = -\frac{\vec{E}e}{m_e} \qquad \vec{v}_e(t) = -\frac{\vec{E}e}{m_e}t$$

However, $\vec{v}_e(t)$ does not increase forever - e^- collides with something & $\vec{v}_e(t)$ is randomized. If τ_{col} is the average time between collisions

$$\langle \vec{v}_e \rangle = \frac{\vec{E}e}{m_e}\frac{\tau_{col}}{2}$$

If the is a density n_e of such conduction electrons per unit volume, a current or flux of charge $\vec{J}(\vec{r}) = +en_e\frac{\vec{E}e}{m_e}\frac{\tau_{col}}{2}$ results

$$= \underbrace{\frac{n_e e^2 \tau_{col}}{2m_e}}_{\sigma \,=\, \text{conductivity}} \vec{E}(\vec{r})$$



Charge passing through δA in time t

$$= -en_eV = -en_e\,\delta A\,\cancel{\cos\theta}\,\langle v \rangle t$$

$$= +en_e\frac{Ee\tau_{col}}{2m_e} \times \delta A\,\cancel{\cos\theta}\,t = \vec{J}\cdot\hat{n}\,\delta A\,t$$

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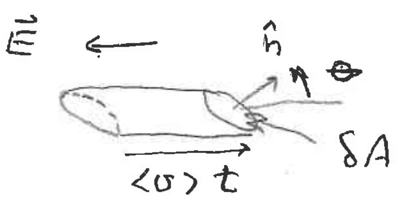

Charge passing through δA in time t

$$= -e n_e V = -e n_e \delta A \cancel{\cos\theta} \langle v \rangle t$$

$$= +e n_e \frac{E e \tau_{col}}{2 m_e} \times \delta A \cancel{\cos\theta}\, t = \vec J \cdot \hat n\, \delta A\, t$$

$\vec J(\vec r) = \sigma \vec E(\vec r)$ is the most basic form of Ohm's law

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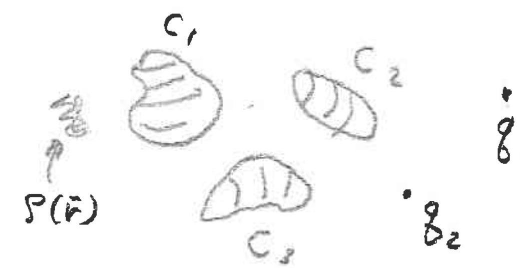
For electrostatics $\vec J = 0$ so Ohm's law requires $\vec E(\vec r) = \frac{1}{\sigma}\vec J = 0$ for $\vec r$ inside a conductor.

Now consider a new type of problem:

Given the location of charges and conductors find $\vec E(\vec r)$ everywhere.



This looks hard because we must determine how the charge is distributed on the conductors to make $\vec E(\vec r) = 0$ inside each and then compute resulting $\vec E(\vec r)$ outside conductors!