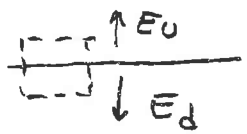
$$E_u = (\sigma_{B_t} + \Delta\sigma)4\pi$$

$$E_d = (\sigma_{B_b} - \Delta\sigma)4\pi$$

How do we determine $\Delta\sigma$?

Expect at a distance system should look like



$\leftarrow \sigma = 10$ esu

with $E_u = E_d$ & $E_u + E_d = 4\pi \cdot 10$ esu

$$E_u = E_d = \frac{10 \text{ esu}}{2\pi}$$

$$\therefore \quad \sigma_{B_t} + \Delta\sigma = 5 \text{ esu}$$

$$\sigma_{B_b} - \Delta\sigma = 5 \text{ esu}$$

$$\frac{5}{13} \times 10 + \Delta\sigma = 5 \text{ esu} \Rightarrow \Delta\sigma = \overbrace{\left(\frac{1}{2} - \frac{5}{13}\right)}^{\frac{3}{26}} \times 10$$

$$\frac{8}{13} \times 10 - \Delta\sigma = 5 \text{ esu} \Rightarrow \Delta\sigma = \left(\frac{8}{13} - \frac{1}{2}\right) \times 10$$

If $E_u \neq E_d$ then $E_u = \frac{5 \text{ esu}}{4\pi} + \Delta E$

$$E_d = \frac{5 \text{ esu}}{4\pi} - \Delta E$$

and we have applied extra uniform $\vec{E} = \Delta E \, \hat{z}$ not specified in problem.

E Capacitance 01/25/2022

1. Given N conducts C_i $1 \leq i \leq N$ each carrying charge Q_i and at the potential Φ_i, how are the Q_i and Φ_i related?

(choose $\lim_{r \to \infty} \Phi(\vec{r}) = 0$)

Solve N different problems:

k^{th} problem has solution $\Phi^{(k)}(\vec{r})$

obeying: ① $\vec{\nabla}^2 \Phi^{(k)}(r) = 0$ outside conductors

② on the i^{th} conductor $\Phi^{i}(r) = \delta_{ik} \Phi_k$

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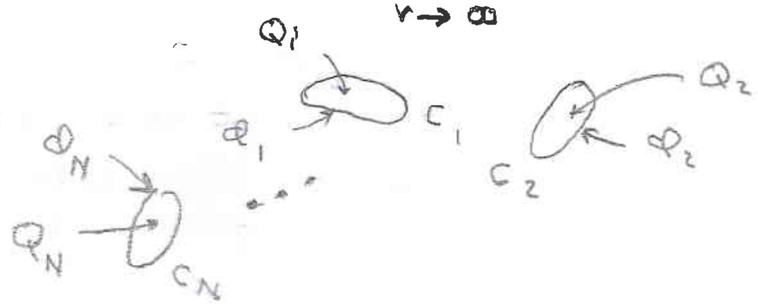

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If $Q_i^{(k)}$ is the charge on the i^{th} conductor for the k^{th} problem,

$$Q_i^{(k)} = \frac{1}{4\pi} \int_{\partial C_i} \vec{E}^{(k)}(\vec{r}) \cdot \hat{n} \, dS$$

$$= \underbrace{\frac{-\frac{1}{4\pi} \int_{\partial C_i} \vec{\nabla} \Phi^{(k)}(\vec{r}) \cdot \hat{n} \, dS}{\Phi_k}}_{\substack{\text{must not} \\ \text{depend on } \Phi_k}} \times \Phi_k$$

$\longrightarrow C_{ik}$

Solve our general problem by superimposing these N problems

$$\Phi(\vec{r}) = \sum_{k=1}^{N} \Phi^{(k)}(\vec{r})$$

$$\Phi(r)\Big|_{\vec{r} \in C_i} = \sum_k \Phi^{(k)}(\vec{r})\Big|_{r \in C_i} = \Phi_i$$

$$Q_i = \sum_{k=1}^{N} Q_i^{(k)} = \sum_k C_{ik} \Phi_k$$

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$$Q_i = \sum_{k=1}^{N} Q_i^{(k)} = \sum_k C_{ik} \varphi_k$$

Thus if we can compute the N^2 coefficients of capacitances C_{ik} the charges Q_i are linearly related to the potentials φ_k

$$Q_i = \sum_{k=1}^{N} C_{ik} \varphi_k$$

2. Find the capacitance of a conducting sphere of radius R:

$$Q_1 = C_{11} \varphi_1$$

$$\varphi(\vec{r}) = \begin{cases} \dfrac{Q}{r} & r \geq R \\ \dfrac{Q}{R} & r \leq R \end{cases}$$

For a hollow sphere,

$$\varphi_1 = \frac{Q_1}{R} \quad \text{or} \quad C_{11} = R$$

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3. Consider two conductors

for C_{ij}

$$\begin{pmatrix} Q_1 \\ Q_2 \end{pmatrix} = \begin{pmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{pmatrix} \begin{pmatrix} \Phi_1 \\ \Phi_2 \end{pmatrix}$$

Consider further the case of practical interest where the system of two conductors is neutral with $Q_1 = -Q_2 = Q$ then

$$\begin{pmatrix} \Phi_1 \\ \Phi_2 \end{pmatrix} = C^{-1}\begin{pmatrix} Q \\ -Q \end{pmatrix} = \frac{1}{C_{11}C_{22} - C_{21}C_{12}} \begin{pmatrix} C_{22} & -C_{12} \\ -C_{21} & C_{11} \end{pmatrix} \begin{pmatrix} Q \\ -Q \end{pmatrix}$$

$$= \frac{1}{C_{11}C_{22} - C_{21}C_{12}} \begin{pmatrix} [C_{22} + C_{12}]\,Q \\ -[C_{21} + C_{11}]\,Q \end{pmatrix}$$

If we use $\Phi = \Phi_1 - \Phi_2$ as the potential difference between the two conducts

$$\Phi = \frac{C_{22} + C_{12} + C_{21} + C_{11}}{C_{22}C_{11} - C_{12}C_{21}}\, Q \equiv \frac{Q}{C}$$

where C is the usual capacitance

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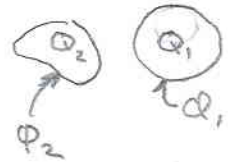

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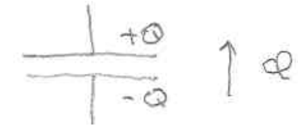
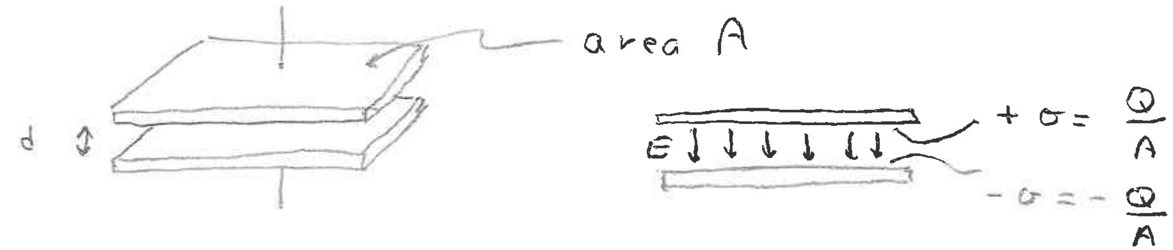
This is a detailed description of a capacitor



4 Examine a parallel plate capacitor



$$E = 4\pi\sigma = \frac{4\pi Q}{A} \qquad \varphi = E d = \frac{4\pi d}{A} Q = \frac{Q}{C}$$

$$C = \frac{A}{4\pi D}$$

The larger A and the smaller d the larger is the charge Q that can be stored for fixed voltage φ

5 Units:

esu $\quad C = \frac{Q}{V} \sim \text{cm}$

SI $\quad C = \frac{Q}{C} \sim \frac{\text{Coulomb}}{\text{Volt}} \equiv \text{Farad}$

$$1\ \text{Farad} = \frac{1\ \text{Coulomb}}{1\ \text{Volt}} = \frac{3\times 10^9\ \text{esu}}{1/300\ \text{statvolt}} = 9\times 10^{11}\ \text{cm}$$

$(2.99792458)^2$

A Farad is large

A cm is small

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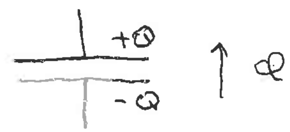

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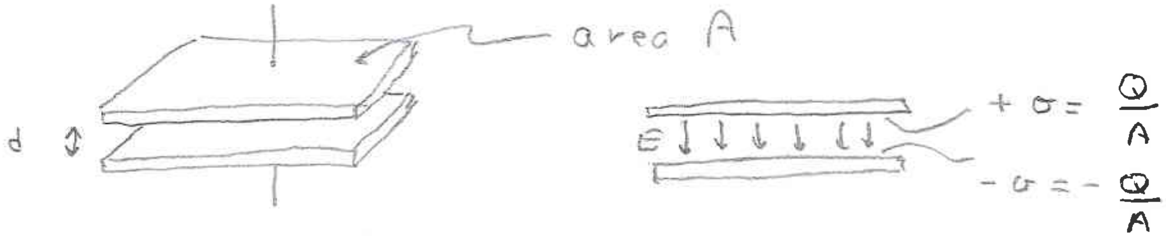

$E = 4\pi\sigma = \frac{4\pi Q}{A}$ $\quad V = Ed = \frac{4\pi d}{A} Q = \frac{Q}{C}$

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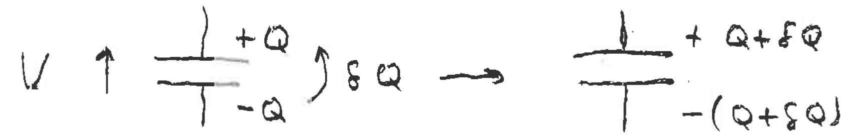
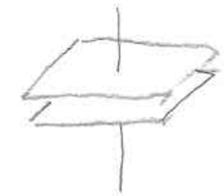
6. Energy stored in a capacitor



$\delta E = V\,\delta Q = \frac{Q}{C}\,\delta Q$

$E = \int_0^E dE' = \int_0^Q \frac{Q}{C}\, dQ = \frac{Q^2}{2C}$

Try out our energy density result

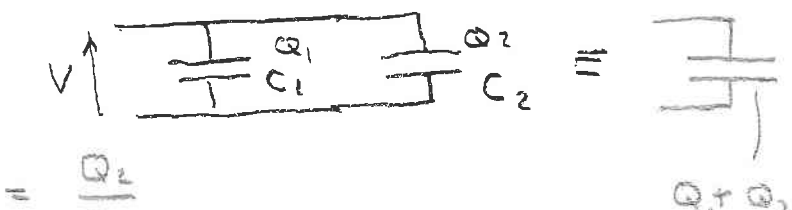


$\text{Energy} = \frac{E^2}{8\pi} Ad = \frac{(4\pi\sigma)^2 Ad}{8\pi}$

$= \frac{1}{2}\,\frac{4\pi d}{A}\, Q^2 = \frac{1}{2}\,\frac{Q^2}{C}$ ✓

7. Combine capacitors

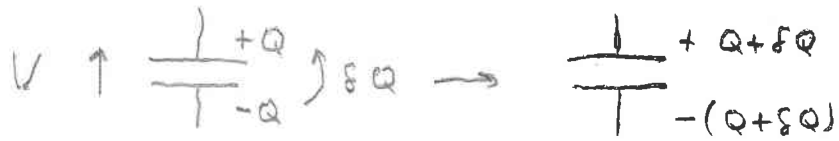
- Parallel



$V = \frac{Q_1}{C_1} = \frac{Q_2}{C_2}$

$C = \frac{Q_1 + Q_2}{V} = C_1 + C_2$

6. Energy stored in a capacitor



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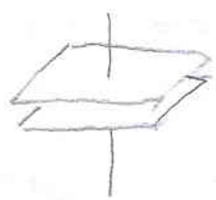

$$\text{Energy} = \frac{E^2}{8\pi} A d = \frac{(4\pi\sigma)^2 A d}{8\pi}$$

$$= \frac{1}{2}\,\frac{4\pi d}{A}\,Q^2 = \frac{1}{2}\,\frac{Q^2}{C} \checkmark$$

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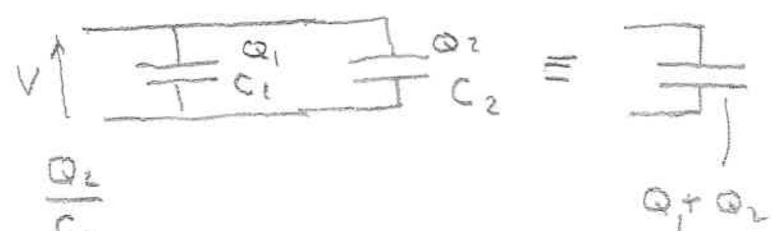

$$V = \frac{Q_1}{C_1} = \frac{Q_2}{C_2}$$

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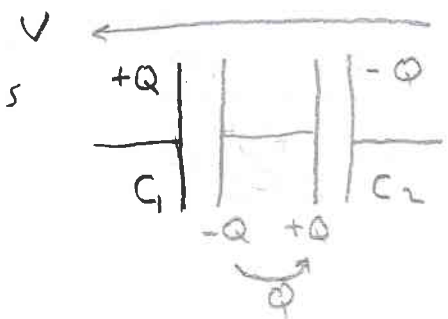
- Series

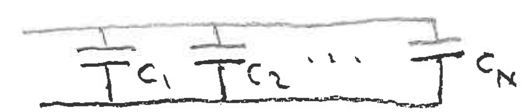


$$V = V_1 + V_2 = \frac{Q}{C_1} + \frac{Q}{C_2} = \frac{Q}{C}$$

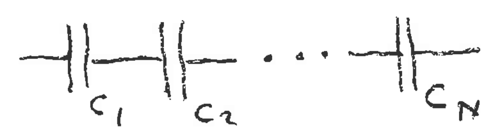
$$\frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{C}$$

$$\text{or} \quad C = \frac{C_1 C_2}{C_1 + C_2}$$

- Easy to generalize to N



$$C = \sum_{i=1}^{N} C_i$$



$$C = \left[\sum_{i=1}^{N} \frac{1}{C_i}\right]^{-1}$$

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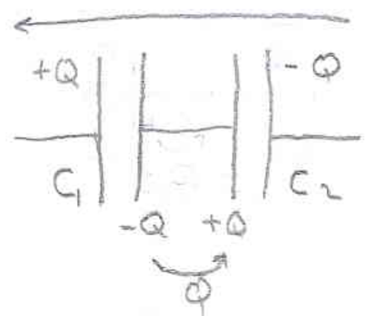

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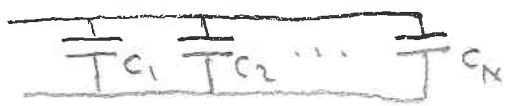

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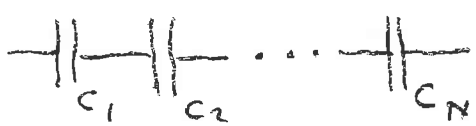

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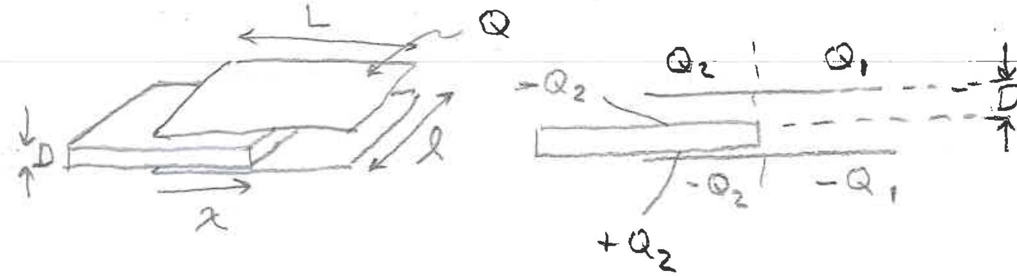
8 Problem: Find force on conducting slab pulled into parallel plate capacitor.



C(x)

$$C_1 = \frac{\ell x}{4\pi D}$$

$$C = C_1 + C_2 \qquad C_2 = \frac{\ell (L-x)}{4\pi (2D)}$$

$$\text{Energy} = \frac{Q^2}{2C} = \frac{1}{2} \frac{Q^2}{\frac{\ell x}{4\pi D} + \frac{\ell(L-x)}{4\pi(2D)}}$$

$$= \frac{4\pi D Q^2/\ell}{x + L}$$

$$\text{force} = -\frac{d}{dx}(\text{Energy}) = +\frac{4\pi D Q^2}{\ell (x+L)^2}$$

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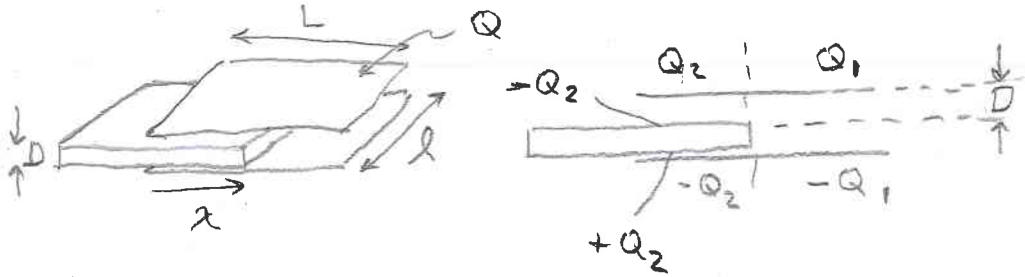
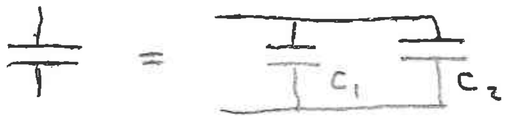


$C(x)$ =



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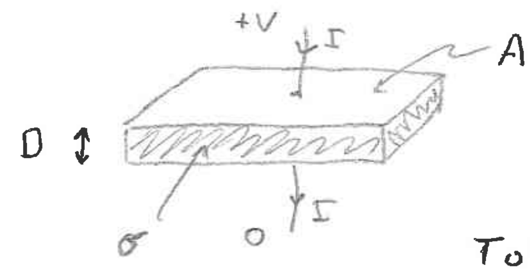
G Ohm's law and circuit theorm

1. Resistors Use $\vec{j} = \sigma \vec{E}$

where $\vec{E}$ is applied field and
$\vec{j}$ is the resulting flux or current of charge (esu/cm²)
σ is conductivity of material

Look at an easy case:



$$ED = V$$

$$j = \sigma E = \sigma \frac{V}{D}$$

Total current

$$I = Aj = \frac{A\sigma}{D} V = \frac{V}{R}$$

were the resistance

$$R = \frac{D}{A\sigma}. \qquad V = IR$$

Ohm's law

Expected behavior

Units $\text{esu} = \frac{\text{esu/cm}}{\text{esu/sec}} = \frac{\text{sec}}{\text{cm}}$

$\text{SI} = \frac{\text{Volt}}{\text{Amp}} = \frac{\text{Volt}}{\text{Coul/sec}} = \text{Ohm}$

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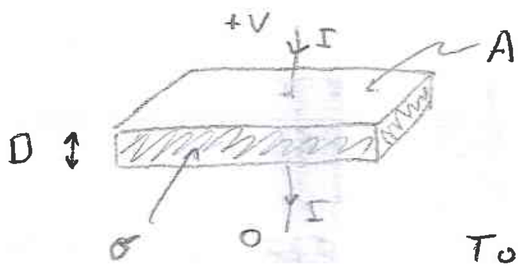

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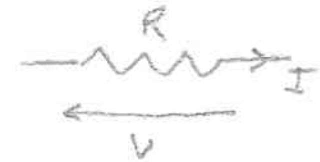
$$= \frac{1}{3 \times 10^{11}} \frac{\text{sec}}{\text{cm}}$$

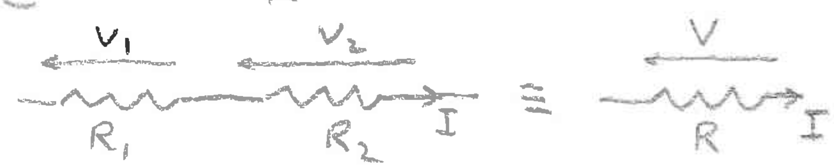
Now the esu unit of $\frac{\text{sec}}{\text{cm}}$ is impractically large, In general use



$V = IR$

2. Circuit theory - introduce discrete elements with two terminals, Connect with high conductivity wires. So far we have

note



$$V_1 = IR_1, \quad V_2 = IR_2$$

$$V = V_1 + V_2 = I\underbrace{(R_1 + R_2)}_{R}$$

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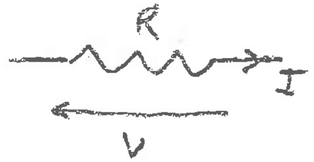

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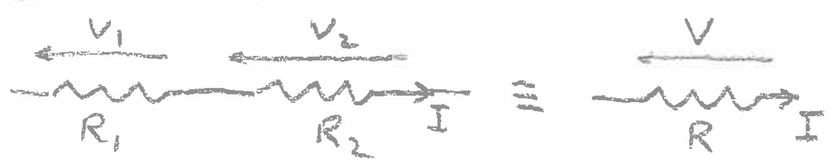

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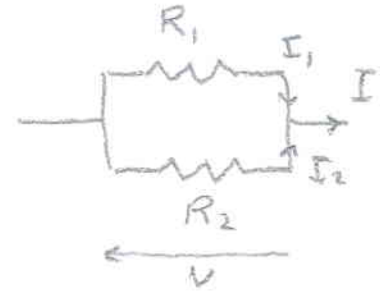
parallel



$I = I_1 + I_2$

$$I = \frac{V}{R_1} + \frac{V}{R_2} = \left[\frac{1}{R_1} + \frac{1}{R_2}\right] V = \frac{V}{R}$$

$$+ \quad \frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$