Physics C2801 — Professor Christ
Final Examination — 12/18/2012

Answer each of the following **four (4)** questions.

Please give a complete description of your method of solution since *partial credit* will be given.

1. Consider a system of two point particles, each of mass m joined by a spring of spring constant k and equilibrium length ℓ. Let $2\,x(t)$ be the distance between the two masses.

   (a) The two masses are constrained to move in a straight line. At $t = 0$, one mass is at rest at the origin, $x(0) = \ell/2$ and the other is moving away from the first with velocity v_0. Find the subsequent motion of each mass. (Let $x_1(t)$ be the location of the particle initially at rest and $x_2(t)$ that for the other.) [10 points]

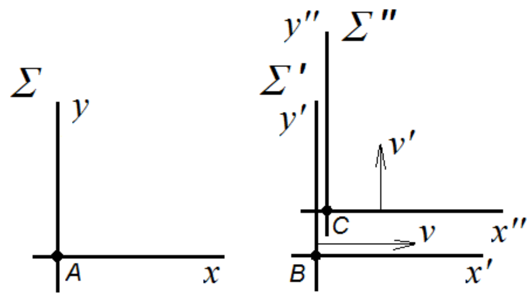
   (b) The same system is now allowed to move in a horizontal plane without friction. If the system is rotating about its center of mass with angular velocity Ω_0, find the distance x_0 between either mass and the center of mass. (Assume that $\Omega_0 < \sqrt{2k/m}$.) [5 points]

   (c) Find the frequency ω for small oscillations about this simple circular motion. [10 points]

2. (a) The K^0 is a particle with mass m_K which has a distinct anti-particle $\overline{K^0}$ with the same mass. Subtle effects of the weak interactions cause a K^0 particle at rest to transform into a $\overline{K^0}$ with a period $T = 1.89\,10^{-10}$ sec. This is called $K^0 - \overline{K^0}$ mixing. Thus, at the time $T/2$ the K^0 will have become a $\overline{K^0}$. In a $K^0 - \overline{K^0}$ mixing experiment a mono-energetic beam of K^0 mesons is created with energy E_K. If a pure sample of $\overline{K^0}$ mesons is desired, how far from the K^0 production point will such a sample be found? Express your answer in terms of the quantities T, E_K and m_K. [10 points]

   (b) A neutral π-meson, π^0 can decay into a photon and an electron-positron pair: $\pi^0 \to \gamma + e^- + e^+$. If the pion has spatial momentum $\vec{p}$, find the smallest and largest energy that can be carried by the photon. [15 points]

3. Consider three coordinate systems Σ, Σ' and Σ''. Label the three spatial origins of these systems, A, B and C. At $t = t' = t'' = 0$ these three points coincide. Further the spatial axes of Σ and Σ' are parallel with their relative velocity $\vec{v}$ pointing along the $+x$ and $-x'$ axes of Σ and Σ' respectively. Likewise the spatial axes of Σ' and Σ'' are parallel with their relative velocity $\vec{v}\,'$ pointing along the $+y'$ and $-y''$ axes of Σ' and Σ'' respectively.

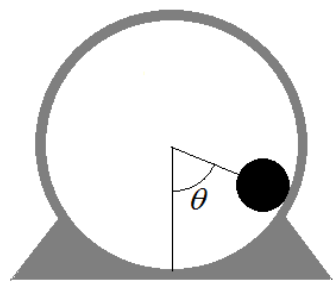


(a) What is the velocity $\vec{v}^{\,C}$ of the point C as observed from the coordinate system Σ? (Find its x and y components.) [10 points]

(b) What is the velocity $\vec{v}^{\,A''}$ of the point A as observed from the coordinate system Σ''? (Find its x'' and y'' components.) [10 points]

(c) Conclude that the axes of Σ and Σ'' are not parallel and determine the angle between them to first order in v'/c for the case that v'/c is small. [5 points]

4. A smaller solid sphere of mass M and radius r rolls without slipping inside a larger fixed sphere of inner radius R. A line between the centers of the two spheres makes a fixed angle θ with a vertical line as shown to the right. The center of mass of the smaller sphere rotates about this vertical direction with constant angular velocity $\vec{\Omega}_{\rm CM}$ pointing upward. The smaller sphere rotates about its center of mass with an angular velocity $\vec{\omega}$ that can be written as the sum of two vectors $\vec{\omega}_C$ and $\vec{\omega}_F$. The angular velocity $\vec{\omega}_C$ points upward and is determined by the constraint that the smaller sphere rolls without slipping inside the larger one. The angular velocity ω_F points in the direction of the line between the centers of the two spheres and is not constrained by the requirement that the small sphere rolls without slipping inside the larger sphere.



(a) Given $|\Omega_{\rm CM}|$, determine $\vec{\omega}_C$. [5 points]

(b) Find the direction and magnitude of the force exerted by the larger sphere on the smaller one. [5 points]

(c) What value must ω_F have if the motion with fixed $\Omega_{\rm CM}$ and θ is to take place as described? [15 points]

Recall that for a sphere the angular momentum and angular velocity are proportional: $\vec{L} = \frac{2}{5}Mr^2\vec{\omega}$. When specifying directions in this problem please use $\hat{z}$ as the unit vector in the vertical direction, $\hat{r}$ as a unit vector pointing from the point of contact to the center of mass of the small sphere and $\hat{t}$ as a unit vector pointing in the direction of the velocity of the center of mass of the small sphere.