Improved version of the explanation of the solution to problem 5 d) given in the review session

5. Two space stations, A and B, are separated by a distance L and are at rest with respect to each other. The two stations lie in the same time zone and have accurately synchronized clocks. A rocket moving at a constant speed v travels from station A to station B. When the rocket pilot passes the first station A, she sets the ship's clock to agree with that of station A. For simplicity assume that they both read zero when the rocket passes station A.
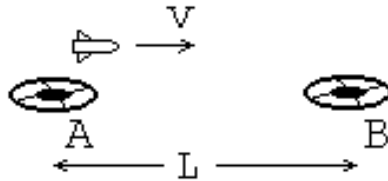
(a) What is the time t_B shown on the clock of station B when the rocket goes by B?

(b) What time is displayed on the ship's clock, t'_B when the rocket passes station B? Explain your result from the perspective of an observer in station B.

(c) Explain the result found in (b) from the perspective of the rocket pilot.

(d) Explain the result found in (a) from the perspective of the rocket pilot.

Solution to 5d):

• The easiest approach is to use Lorentz transformation to change from rocket coordinates to station coordinates. The rocket pilot assigns the coordinates $(x = 0, t = L/(v\gamma))$ to the event of the rocket reaching station B. The coordinates of that event in the station system are $(x = L, t_B)$. The rocket pilot can use the equation:

$$t_B = \gamma(t + vx/c^2) = \gamma L/(v\gamma) = L/v$$

to determine t_B and gets an answer in agreement with the result in a).

• A second approach, not well explained during the review session, goes as follows:

  – When the rocket passes station A both the rocket clock and the station A clock read zero.

  – While the station A observer says that the clocks A and B both read 0 simultaneously, the rocket pilot says that the station B clock 0 reads zero at an earlier time, $-\gamma Lv/c^2$ (our old result for the loss of simultaneity in a moving system).

  – Thus, the rocket observer says that the clock on station B reads zero at the time $-\gamma Lv/c^2$ and must have run an additional time $L/(v\gamma)$ when the rocket finally reaches station B.

  – Since this moving clock runs slow, it should read this total elapsed time determined by the rocket pilot, since it read zero, divided by γ:

$$
\begin{aligned}
t_B &= [\gamma Lv/c^2 + L/(v\gamma)]/\gamma \\
&= L/v[(v/c)^2 + 1/\gamma^2] = L/v
\end{aligned}
$$

as obtained above.