# Supplementary Appendix for "Of Candidates and Character"

B. Douglas Bernheim* Navin Kartik†

November 2, 2010

In this Supplementary Appendix, we formally study an infinitely repeated version of the baseline model, in which insiders are infinite-lived and discount the future at the rate $\delta \in (0,1)$.[1] We assume infinite repetitions for two reasons. First, the impact of a future opportunity to seek re-election on a prospective candidate's decision to run for office in the first place depends on the expected quality of future opponents, which should in turn depend on their opportunities to run as incumbents. Assuming a fixed horizon amounts to imposing a single-term limit as of some point in the future. Second, the infinite horizon case turns out to be simpler in some important ways,[2] and yields some sharper results.

We will assume that governors can hold office for a maximum of two terms, postponing a discussion of longer terms until after presenting the main result. We will also assume that, when an incumbent is eligible to run, she must announce her candidacy before other insiders make their decisions. This sequencing of entry choices is not material but simplifies some of the analysis. Finally, as mentioned earlier, we assume that the electorate necessarily knows the character of an incumbent governor, which is unavoidably revealed in office. Though somewhat stark, the latter assumption captures the essence of the phenomenon we seek to study.

Our analysis will allow for the possibility that a governor's compensation differs across her first and second terms. Back-loading compensation into the second term can rein-

*Department of Economics, Stanford University, and Research Associate, National Bureau of Economic Research. Email: bernheim@stanford.edu.

†Department of Economics, Columbia University. Email: nkartik@columbia.edu.

[1] Because we focus below on Markov-Perfect Equilibria, our analysis would be essentially unchanged if we assumed that each insider was finite-lived. The assumption of infinite-lived agents is for convenience only.

[2] For example, the existence of pure strategy equilibria is problematic with a finite horizon, but not with an infinite horizon. An infinite horizon also allows us to exploit the stationary structure of the problem.

force the incentives for advantageous selection that arise with opportunities for re-election. Henceforth, we will use $s_t$ to denote the governor's compensation in her $t$-th term of office.

Artificial results can arise in such settings through two separate channels: history dependence and out-of-equilibrium beliefs. With respect to history dependence, the infinite horizon creates scope for sustaining a wide variety of outcomes. We therefore focus on equilibria that employ Markovian strategies and beliefs. That is, we allow the entry choice of an incumbent to depend only on her own characteristics, and the entry choice of a non-incumbent insider to depend only on the presence and characteristics of an incumbent candidate. For voters, we allow beliefs about the composition of the non-incumbent candidate pool to depend only on the presence and characteristics of an incumbent candidate, and on the number of non-incumbent candidates. We also insist that the Markovian strategies constitute equilibria in every proper subgame, and refer to such equilibrium as *Markov-perfect*.[3]

The freedom to specify any out-of-equilibrium belief also create scope for strange equilibria. For example, one can construct an equilibrium wherein an incumbent Scoundrel wins while an incumbent of higher quality loses, both with probability one.[4] We therefore restrict attention to equilibria that satisfy a simple and plausible belief restriction, which we now describe.

In a Markov-perfect equilibrium, both choices and voters' beliefs about the composition of the candidate pool can depend on state variables. What follows should be interpreted as being conditional upon a particular state, but for simplicity we suppress that conditionality in the notation. Given the incumbent's quality and entry decision as well as continuation strategies, there is some set of non-incumbent candidates, $\mathcal{N}$, who enter with positive probability, of whom members of some set $\mathcal{N}_1$ enter with probability one. Thus, the realized number of non-incumbent candidates can be any integer between $|\mathcal{N}_1|$ and $|\mathcal{N}|$ (which may be infinite); realizations smaller than $|\mathcal{N}_1|$ and larger than $|\mathcal{N}|$ lie off the equilibrium path. For all $i \in \mathcal{N}_1$, let $\rho_W^i$ be the probability of winning that renders $i$ indifferent between entering and not entering, given the continuation equilibria that would follow each choice. Also, for all types $(a, h) \in [0, 1]^2$, let $\rho_U(a, h)$ be the probability of winning that would render one additional insider of type $(a, h)$ indifferent between entering and not entering, once again given the continuation equilibria.

[3]Our usage of this term is slightly non-standard, in that Markov-perfection is typically used with reference to games of complete information. Here, voters have incomplete information concerning candidates' characteristics.

[4]An incumbent Scoundrel can win with probability one if voters assume that all who enter against him are also Scoundrels.

**Belief Restriction:** For any realized number of candidates $N < |\mathcal{N}_1|$, voters assume that the $|\mathcal{N}_1| - N$ members of $\mathcal{N}_1$ with the largest values of $\rho_W^i$ did not enter. For any realized number of candidates $N > |\mathcal{N}|$ , voters assume that all members of $\mathcal{N}$ entered along with $N - |\mathcal{N}|$ additional insiders whose characteristics minimize $\rho_U(a, h)$ on $[0, 1]^2$.

The restriction attributes zero-probability events to deviations by the smallest possible number of insiders, and to the particular insiders with the greatest incentives (or smallest disincentives) to make those deviations. Thus, it is in the spirit of other common belief restrictions (e.g. Cho and Kreps, 1987; Bagwell and Ramey, 1991). We do not mean to suggest that all equilibria that violate this restriction are *necessarily* problematic. Rather, by imposing this restriction, we insuring against results that are driven by contrived or otherwise unrealistic out-of-equilibrium beliefs.

Our main result for this infinitely-repeated game is:

**Theorem 1.** *For any $k > 0$, provided $s_2$ is sufficiently large, for every $y^* \in [Y(1, 0 \mid \sigma), y^{\max}]$, there exists a pure strategy Markov-perfect equilibrium satisfying the Belief Restriction for which the quality of governance is $y^*$ in all periods.*

The theorem establishes several important principles. Allowing governors to run for a second term can attract high-quality candidates and eliminate the variability of governance (noted in Sections 4.2 and 4.4). If second-term compensation is sufficiently high, there are equilibria in which only citizens of the highest character run for office. However, even incumbency with highly back-loaded compensation does not *guarantee* the highest levels of quality. There are also equilibria in which governors of much lower quality are reelected (though average quality is still higher than with one-term limits).[5]

The equilibria that give rise to the outcomes described in Theorem 1 share a simple structure: all candidates are (and are believed to be) of quality $y^*$, and incumbent candidates win if and only if their quality is no lower than $y^*$. Intuitively, supposing candidates win re-election if and only if they are of quality $y^*$ or greater (with $y^* \geq Y(1, 0 \mid \sigma)$), then the insiders with the greatest incentive to run for office in the first place have quality $y^*$

[5]Furthermore, there are likely other equilibria satisfying the Belief Restriction that do not belong to the class we examine in proving the Theorem; we have not ruled out the possibility that some involve even lower quality or substantial variability in the quality of governance. However, Theorem **??** implies that any equilibrium yielding governance quality at or below the level achieved in the one-period model would have to be rather strange, e.g., involving probabilities of reelection that are decreasing in quality.

(candidates of lower quality benefit from only a single term, while candidates of higher quality forego a portion of the benefits potentially received from special interests).[6] Thus, any re-election quality threshold $y^*$ between $Y(1, 0 \mid \sigma)$ and $y_{\max}$ bootstraps itself into an equilibrium. Notice also that backloading pay to a greater degree does not discriminate in favor of those with higher quality among the set of insiders whose quality exceeds $y^*$; hence, it cannot improve selection, and only serves to reinforce the disincentives to enter among insiders whose quality falls below any given reelection threshold, $y^*$.

Theorem 1 pertains to a two-term limit. With a one-term limit, a Markov-perfect equilibrium simply repeats the equilibria of the one-period model. Thus, for small $k$, switching from a two-term limit to a one-term limit reduces the quality of governance. In contrast, a two-term limit does not inherently reduce the quality of governance relative to limits of three or more terms. In this model, extending the limit beyond two terms simply increases the reward for those of sufficient quality to win re-election. The same end can be accomplished with a two-term limit through higher second-term compensation.[7]

*Proof of Theorem 1.* **Step 1:** We begin by constructing a class of equilibria; subsequently we verify that these equilibria have the desired properties.

Select some $y^* \in [Y(1, 0 \mid \sigma), y_{\max}]$. Construct insiders' strategies as follows:

(s-i) If there is an incumbent, she runs for re-election if and only if $y^I \geq y^*$.

(s-ii) If there is an incumbent with $y^I > y^*$ who runs, no non-incumbent candidates enter.

(s-iii) If no incumbent runs (possibly because there is no incumbent), or if there is an incumbent with $y^I < y^*$ who runs, $N^* \geq 2$ non-incumbent candidates enter, each with characteristics $(a^*, h^*)$ and quality $y^*$ (where we define $N^*$, $a^*$, and $h^*$ below).

Construct voters beliefs about non-incumbent candidates as follows:

(b-i) If there is an incumbent with $y^I \geq y^*$ who runs opposed, voters believe that all non-incumbent candidates have characteristics $\left(\widehat{a}(y^I), \widehat{h}(y^I)\right)$ and quality $y^*$ (where we define the functions $\left(\widehat{a}, \widehat{h}\right)$ below).

[6] For $y < Y(1, 0 \mid \sigma)$, Sell-Outs may have the greatest incentive to run for office, so the argument breaks down.

[7] Plainly, if there is a binding upper bound on the level of compensation that can be given in any one period, longer term limits may be useful.

(b-ii) If there is no incumbent, an incumbent who does not run, or an incumbent with $y^I < y^*$ who does run, then voters believe that any and all non-incumbent candidates have characteristics $(a^*, h^*)$ and quality $y^*$.

Observe that the beliefs are consistent with the strategies on the equilibrium path. If $y^I \geq y^*$ then the incumbent runs (by (s-i)) and no non-incumbents enter (by (s-ii)); hence beliefs about non-incumbent candidates are not relevant. If $y^I < y^*$ then the incumbent does not run (by (s-i)) and $N^*$ non-incumbents enter (by (s-iii)), each with characteristics $(a^*, h^*)$ and quality $y^*$, which is consistent with beliefs (according to (b-ii)).

Next observe that with these strategies and beliefs, elections play out as follows:

(e-i) If $y^I \geq y^*$ and the incumbent runs for re-relection, whether opposed or unopposed, she wins. (When opposed, voters believe that the non-incumbent candidates are of quality $y^*$ by (b-i), so the incumbent prevails.)

(e-ii) If $y^I < y^*$ and the incumbent runs for re-election, she is opposed (by s-iii) and loses (because voters believe that the non-incumbent candidates are of quality $y^*$ by (b-ii)). Notice that she loses even if the number of opposing candidates is not $N^*$.

(e-iii) If the incumbent does not run for re-election, a non-incumbent candidate with characteristics $(a^*, h^*)$ and quality $y^*$ wins (by (s-iii)).

Now we define $N^*$, $(a^*, h^*)$, and $\left(\widehat{a}, \widehat{h}\right)$. Consider the locus $C^*$ defined by the condition $(a, h) \in C^*$ if and only if $Y(a, h \mid \sigma) = y^*$. Also define

$$\Delta^I(a, h, y) := u^G(a, h \mid \sigma, s_1) + \delta u^G(a, h \mid \sigma, s_2) - a(y + \delta y^*).$$

Let

$$\left(\widehat{a}(y), \widehat{h}(y)\right) := \arg \max_{(a,h) \in C^*} \Delta^I(a, h, y).$$

If the maximum above is not uniquely defined for some $y$, take an arbitrary selection. Also, let

$$(a^*, h^*) := \left(\widehat{a}(y^*), \widehat{h}(y^*)\right),$$

and let

$$N^* := \left\lfloor \frac{\Delta^I(a^*, h^*, y^*)}{k}, \right\rfloor$$

(where $\lfloor x \rfloor$ denotes the "floor" function, which returns the largest integer less than or equal to $x$). Notice that for $s_2$ sufficiently large, $N^* \geq 2$ as required.

**Step 2:** Next we verify that no insider has an incentive to deviate from the strategies identified in Step 1. There are a number of cases to consider.

(i) An incumbent with characteristics $(a^I, h^I)$ and quality $y^I \geq y^*$. If she does not run, the quality of governance is $y^*$ in the current period (by (e-iii)) and all subsequent periods (by (e-i) through (e-iii)). If she runs, she wins re-election with probability one (by (e-i)), and the quality of governance is $y^*$ in all subsequent periods. Hence, she is willing to run if and only if

$$u^G\left(a^I, h^I \mid \sigma, s_2\right) - a^I y^* \geq 0,$$

which plainly holds for $s_2$ sufficiently large.

(ii) An incumbent with characteristics $(a^I, h^I)$ and quality $y^I < y^*$. If she runs, she looses; whether she runs or not, the quality of governance is $y^*$ in all periods (by (e-i) through (e-iii)). Therefore, with $k > 0$, she prefers not to run.

(iii) A non-incumbent non-candidate when there is an incumbent with $y^I \geq y^*$, who runs. If the non-incumbent non-insider runs, she loses; whether she runs or not, the quality of governance is $y^*$ in all periods. Therefore, with $k > 0$ she prefers not to run.

(iv) A non-incumbent non-candidate when there is no incumbent, there is an incumbent who does not run, or there is an incumbent with $y^I < y^*$ who runs. First consider a non-incumbent non-candidate with characteristics $(a', h')$ and quality $y' \geq y^*$. If such a candidate runs and wins, she will also win re-election, so her payoff is

$$u^G\left(a', h' \mid \sigma, s_1\right) + \delta u^G\left(a', h' \mid \sigma, s_2\right) + (1 + a')\frac{\delta^2}{1-\delta}y^*.$$

If she does not run, her payoff is $(1 + a^*)\frac{1}{1-\delta}y^*$. With $N^* + 1$ candidates, she therefore prefers not to run when

$$\frac{1}{N^* + 1}\Delta^I(a', h', y^*) \leq k. \tag{1}$$

As we have previously noted, $u^G$ (and hence $\Delta^I$) is strictly decreasing in $h$. Thus, within the set of candidate characteristics $\{(a', h') \mid Y(a', h' \mid \sigma) \geq y^*\}$ (for which $C^*$ is the lower boundary), the left-hand side of (1) is maximized for $(a', h') = (a^*, h^*)$ (which by construction maximizes that expression on $C^*$). It follows that (1) is satisfied within $\{(a', h') \mid Y(a', h' \mid \sigma) \geq y^*\}$ provided $\frac{1}{N^*+1}\Delta^I(a^*, h^*, y^*) \leq k$. But by construction, $N^*+1 > \frac{\Delta^I(a^*,h^*,y^*)}{k}$, so the preceding inequality holds.

Next consider a non-incumbent non-candidate with characteristics $(a', h')$ and quality

$y' < y^*$. If such a candidate runs and wins, she will not win re-election, so her payoff is

$$u^G(a', h' \mid \sigma, s_1) + (1 + a')\frac{\delta}{1-\delta}y^*.$$

If she does not run, her payoff is $(1 + a^*)\frac{1}{1-\delta}y^*$. With $N^* + 1$ candidates, she therefore prefers not to run when

$$\frac{1}{N^* + 1}\left[u^G(a', h' \mid \sigma, s_1) - (1 + a')y^*\right] \leq k. \tag{2}$$

But with $s_2$ sufficiently large,

$$\Delta^I(a^*, h^*, y^*) > u^G(a', h' \mid \sigma, s_1) - (1 + a')y^*.$$

Thus, (2) follows from $\frac{1}{N^*+1}\Delta^I(a^*, h^*, y^*) \leq k$.

(v) A non-incumbent candidate when there is no incumbent, there is an incumbent who does not run, or there is an incumbent with $y^I < y^*$ who runs. By (s-iii), all such candidates have characteristics $(a^*, h^*)$ and are of quality $y^*$. If one such candidate runs and wins, she will also win re-election, so her payoff is

$$u^G(a^*, h^* \mid \sigma, s_1) + \delta u^G(a^*, h^* \mid \sigma, s_2) + (1 + a^*)\frac{\delta^2}{1-\delta}y^*.$$

If she does not run, her payoff is $(1 + a^*)\frac{1}{1-\delta}y^*$. With $N^*$ candidates, she therefore prefers to run when $\frac{1}{N^*}\Delta^I(a^*, h^*, y^*) \geq k$. But by construction, $N^* \leq \frac{\Delta^I(a^*,h^*,y^*)}{k}$, so the preceding inequality holds.

**Step 3:** Finally we verify that beliefs satisfy the Belief Restriction.

We begin with (b-i). Suppose there is an incumbent with $y^I \geq y^*$ who runs. Here, $|\mathcal{N}| = 0$, so we are concerned with beliefs when entry unexpectedly occurs. In the continuation (with no entry against the incumbent), the quality of governance would be $y^I$ for a single period, followed by $y^*$ in all subsequent periods. For an insider with characteristics $(a, h)$ such that $Y(a, h \mid \sigma) \geq y^*$, we therefore have

$$\rho_U(a, h) = \frac{k}{\Delta^I(a, h, y^I)},$$

because such an insider, if victorious, would be re-elected. Notice that $\left(\widehat{a}(y^I), \widehat{h}(y^I)\right)$ by construction maximizes $\Delta^I(a, h, y^I)$ and therefore minimizes $\rho_U(a, h)$ for $(a, h)$ satisfying $Y(a, h \mid \sigma) \geq y^*$. Moreover, for an insider with characteristics $(a, h)$ such that $Y(a, h \mid \sigma) < y^*$,

$$\rho_W(a, h) = \frac{k}{u^G(a, h \mid \sigma, s_1) - (1 + a)y^I},$$

because such an insider, if victorious, would not be re-elected. But with $s_2$ sufficiently large, $\Delta^I(\widehat{a}(y^I), \widehat{h}(y^I), y^I) > u^G(a, h \mid \sigma, s_1) - (1 + a)y^*$. Thus, $\left(\widehat{a}(y^I), \widehat{h}(y^I)\right)$ minimizes $\rho_W^n(a, h)$ for $(a, h) \in [0, 1]^2$, as required.

Now consider (b-ii). Suppose there is no incumbent, an incumbent who does not run, or an incumbent with $y^I < y^*$ who does run. Here $|\mathcal{N}| = |\mathcal{N}_1| = N^*$, so we are concerned with beliefs when the realized number of firms, $N$, is both greater than and less than $N^*$. The case of $N < N^*$ is simple because the pool of equilibrium entrants is homogeneous, consisting of insiders with characteristics $(a^*, h^*)$; hence, any subset must also consist of insiders with characteristics $(a^*, h^*)$. For the case of $N > N^*$, the argument parallels the one given for (b-i), except that the quality of governance in the continuation (conditional on the presence of an incumbent as well as the incumbent's type and entry choice) is $y^*$ for the current period, rather than $y^I$, so that $y^*$ replaces $y^I$ throughout. □

# References


**Bagwell, Kyle and Garey Ramey**, "Oligopoly Limit Pricing," *RAND Journal of Economics*, Summer 1991, *22* (2), 155–172. 3

**Cho, In-Koo and David Kreps**, "Signaling Games and Stable Equilibria," *Quarterly Journal of Economics*, 1987, *102* (2), 179–221. 3