Supplementary Appendix for
"Optimal Contracts for Experimentation"

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SA.1. Proof of Proposition 1

We prove the result more generally for contracts with lockouts. Fix a contract $\mathbf{C} = (\Gamma, W_0, \boldsymbol{b}, \boldsymbol{l})$. The result is trivial if $\Gamma = \emptyset$, so assume $\Gamma \neq \emptyset$. Let $T = \max \Gamma$. For any period $t \in \Gamma$ with $t < T$, define the smallest successor period in Γ as $\sigma(t) = \min\{t' : t' > t, t' \in \Gamma\}$; moreover, let $\sigma(0) = \min \Gamma$.

Given any action profile for the agent, the agent's time-zero expected discounted payoff when his type is $\theta \in \{L, H\}$ and the principal's time-zero expected discounted payoff only depend upon a contract's induced vector of discounted transfers, say $(\tau_t)_{t\in\Gamma}$ when success is obtained in period t and on the discounted transfer when there is no success. Hence, it suffices to construct a penalty contract, $\widehat{\mathbf{C}}$, and bonus contract, $\widetilde{\mathbf{C}}$, that induce the same such vector of transfers as $\mathbf{C}$.

To this end, define the penalty contract $\widehat{\mathbf{C}} = (\Gamma, \widehat{W}_0, \widehat{\boldsymbol{l}})$ as follows:

(a) For any t such that $t < T$ and $t \in \Gamma$, $\widehat{l}_t = l_t - b_t + \delta^{\sigma(t)-t} b_{\sigma(t)}$.

(b) $\widehat{l}_T = l_T - b_T$.

(c) $\widehat{W}_0 = W_0 + \delta^{\sigma(0)} b_{\sigma(0)}$.

Define the bonus contract $\widetilde{\mathbf{C}} = (\Gamma, \widetilde{W}_0, \widetilde{\boldsymbol{b}})$ as follows:

(a) For any $t \in \Gamma$, $\widetilde{b}_t = b_t - \sum\limits_{s \geq t, s\in\Gamma} \delta^{s-t} l_s$.

(b) $\widetilde{W}_0 = W_0 + \sum\limits_{t\in\Gamma} \delta^t l_t$.

Consider first the discounted transfer induced by each of these three contracts if success is not obtained. For $\mathbf{C}$, it is $W_0 + \sum_{t\in\Gamma} \delta^t l_t$. For $\widehat{\mathbf{C}}$, it is

$$\widehat{W}_0 + \sum_{t\in\Gamma} \delta^t \widehat{l}_t = W_0 + \delta^{\sigma(0)} b_{\sigma(0)} + \sum_{t\in\Gamma, t<T} \delta^t \left(l_t - b_t + \delta^{\sigma(t)-t} b_{\sigma(t)}\right) + \delta^T (l_T - b_T) = W_0 + \sum_{t\in\Gamma} \delta^t l_t,$$

where the first equality follows from the definition of $\widehat{\mathbf{C}}$ and the second from algebraic simplification. For $\widetilde{\mathbf{C}}$, since there are no penalties, the corresponding discounted transfer is just $\widetilde{W_0} = W_0 + \sum_{t\in\Gamma} \delta^t l_t$. Hence, all three contracts induce the same transfer in the event of no success.

Next, for any $s \in \Gamma$, consider a success obtained in period s. The discounted transfer in this event in $\mathbf{C}$ is $W_0 + \sum_{t\in\Gamma, t<s} \delta^t l_t + \delta^s b_s$. For $\widehat{\mathbf{C}}$, since there are no bonuses, it is

$$\widehat{W_0} + \sum_{t\in\Gamma, t<s} \delta^t \widehat{l_t} = W_0 + \delta^{\sigma(0)} b_{\sigma(0)} + \sum_{t\in\Gamma, t<s} \delta^t \left(l_t - b_t + \delta^{\sigma(t)-t} b_{\sigma(t)}\right) = W_0 + \sum_{t\in\Gamma, t<s} \delta^t l_t + \delta^s b_s,$$

where again the first equality uses the definition of $\widehat{\mathbf{C}}$ and the second follows from simplification. For $\widetilde{\mathbf{C}}$, since there are no penalties, the corresponding discounted transfer is

$$\widetilde{W_0} + \delta^s \widetilde{b}_s = W_0 + \sum_{t\in\Gamma} \delta^t l_t + \delta^s \left(b_s - \sum_{t\geq s, t\in\Gamma} \delta^{t-s} l_s \right) = W_0 + \sum_{t\in\Gamma, t<s} \delta^t l_t + \delta^s b_s,$$

where again the first equality is by definition of $\widetilde{\mathbf{C}}$ and the second from simplification. Hence, all three contracts induce the same transfer in the event of success in any period $s \in \Gamma$.

SA.2. Proof of Proposition 2

We use a monotone comparative statics argument. Recall expression (B.20), which was the portion of the principal's objective that involves a stopping time for the low type, T:

$$\begin{aligned} V(T, \beta_0, \mu_0, c, \delta, \lambda^L, \lambda^H) \quad := \quad & (1-\mu_0)\left[\beta_0 \sum_{t=1}^{T} \delta^t \left(1-\lambda^L\right)^{t-1}\left(\lambda^L - c\right) - (1-\beta_0)\sum_{t=1}^{T} \delta^t c\right] \\ & -\mu_0\beta_0 \left\{ \begin{array}{l} \sum_{t=1}^{T} \delta^t \bar{l}_t^L(T) \left[\left(1-\lambda^H\right)^t - \left(1-\lambda^L\right)^t\right] \\ -\sum_{t=1}^{T} \delta^t c \left[\left(1-\lambda^H\right)^{t-1} - \left(1-\lambda^L\right)^{t-1}\right] \end{array} \right\}, \end{aligned}$$

where $\bar{l}_t^L(T)$ is given by (6) in Theorem 3. The second-best stopping time, $\bar{t}^L$, is the T that maximizes $V(T, \cdot)$.[1] To establish the comparative statics of $\bar{t}^L$ with respect to the parameters, we show that $V(T, \cdot)$ has increasing or decreasing differences in T and the relevant parameter.

Substituting $\bar{l}_t^L(T)$ from (6) into $V(\cdot)$ above yields

$$V(T, \beta_0, \mu_0, c, \delta, \lambda^L, \lambda^H) = (1-\mu_0)\left[\beta_0 \sum_{t=1}^{T} \delta^t \left(1-\lambda^L\right)^{t-1}\left(\lambda^L - c\right) - (1-\beta_0)\sum_{t=1}^{T} \delta^t c\right]$$

[1] While the maximizer is generically unique, recall that if multiple maximizers exist we select the largest one.

$$
-\mu_0 \left\{ \begin{array}{l} -c \sum_{t=1}^{T-1} \delta^t \left(1-\delta\right) \frac{\beta_0 \left(1-\lambda^L\right)^{t-1}+1-\beta_0}{\lambda^L (1-\lambda^L)^{t-1}} \left[\left(1-\lambda^H\right)^t - \left(1-\lambda^L\right)^t \right] \\ -c\delta^T \frac{\beta_0 \left(1-\lambda^L\right)^{T-1}+1-\beta_0}{\lambda^L (1-\lambda^L)^{T^L-1}} \left[\left(1-\lambda^H\right)^T - \left(1-\lambda^L\right)^T \right] \\ -\beta_0 \sum_{t=1}^{T} \delta^t c \left[\left(1-\lambda^H\right)^{t-1} - \left(1-\lambda^L\right)^{t-1} \right] \end{array} \right\}. \tag{SA.1}
$$

After some algebraic manipulation, we obtain

$$
\begin{aligned} & V(T+1, \beta_0, \mu_0, c, \delta, \lambda^L, \lambda^H) - V(T, \beta_0, \mu_0, c, \delta, \lambda^L, \lambda^H) \\ &= \delta^{T+1} \left\{ \begin{array}{l} (1-\mu_0) \left[\beta_0 \left(1-\lambda^L\right)^T \left(\lambda^L - c\right) - (1-\beta_0) c \right] \\ -\mu_0 c \frac{\beta_0 \left(1-\lambda^L\right)^T + 1 - \beta_0}{(1-\lambda^L)^T \lambda^L} \left(1-\lambda^H\right)^T \left(\lambda^H - \lambda^L\right) \end{array} \right\}. \end{aligned} \tag{SA.2}
$$

(SA.2) implies that $V(T, \beta_0, \mu_0, c, \delta, \lambda^L, \lambda^H)$ has increasing differences in (T, β_0), because

$$
\frac{\partial}{\partial \beta_0} \left[V(T+1, \beta_0, \cdot) - V(T, \beta_0, \cdot) \right] = \delta^{T+1} \left\{ \begin{array}{l} (1-\mu_0) \left[\left(1-\lambda^L\right)^T \left(\lambda^L - c\right) + c \right] \\ +\mu_0 c \frac{1-\left(1-\lambda^L\right)^T}{(1-\lambda^L)^T \lambda^L} \left(1-\lambda^H\right)^T \left(\lambda^H - \lambda^L\right) \end{array} \right\} > 0.
$$

It thus follows that $\bar{t}^L$ is increasing in β_0. Similarly, (SA.2) also implies

$$
\frac{\partial}{\partial c} \left[V(T+1, c, \cdot) - V(T, c, \cdot) \right] = \delta^{T+1} \left\{ \begin{array}{l} -(1-\mu_0) \left[\beta_0 \left(1-\lambda^L\right)^T + (1-\beta_0) \right] \\ -\mu_0 \frac{\beta_0 \left(1-\lambda^L\right)^T + 1 - \beta_0}{(1-\lambda^L)^T \lambda^L} \left(1-\lambda^H\right)^T \left(\lambda^H - \lambda^L\right) \end{array} \right\} < 0,
$$

and hence $\bar{t}^L$ is decreasing in c.

To obtain the comparative static of $\bar{t}^L$ in μ_0, we compute

$$
\frac{\partial}{\partial \mu_0} \left[V(T+1, \mu_0, \cdot) - V(T, \mu_0, \cdot) \right] = \delta^{T+1} \left\{ \begin{array}{l} -\left[\beta_0 \left(1-\lambda^L\right)^T \left(\lambda^L - c\right) - (1-\beta_0) c \right] \\ -c \frac{\beta_0 \left(1-\lambda^L\right)^T + 1 - \beta_0}{(1-\lambda^L)^T \lambda^L} \left(1-\lambda^H\right)^T \left(\lambda^H - \lambda^L\right) \end{array} \right\}. \tag{SA.3}
$$

Recall that the first-best stopping time t^L is such that $\frac{\beta_0 \left(1-\lambda^L\right)^{t^L-1}}{\beta_0 (1-\lambda^L)^{t^L-1} + 1 - \beta_0} \lambda^L \geq c$, which is equivalent to $\beta_0 \left(1-\lambda^L\right)^{t^L-1} \left(\lambda^L - c\right) - (1-\beta_0)\, c \geq 0$. Thus, for $T+1 \leq t^L$,

$$
\beta_0 \left(1-\lambda^L\right)^T \left(\lambda^L - c\right) - (1-\beta_0)\, c \geq 0. \tag{SA.4}
$$

Combining (SA.3) and (SA.4) implies

$$\frac{\partial}{\partial \mu_0}\left[V(T+1,\mu_0,\cdot)-V(T,\mu_0,\cdot)\right] \leq -\delta^{T+1} c \frac{\beta_0\left(1-\lambda^L\right)^T+1-\beta_0}{\left(1-\lambda^L\right)^T \lambda^L}\left(1-\lambda^H\right)^T\left(\lambda^H-\lambda^L\right)<0.$$

It follows that $\bar{t}^L$ is decreasing in μ_0.

We next consider the comparative statics of $\bar{t}^L$ with respect to λ^L and λ^H. For λ^L, note that since $T+1 \leq t^L$ and the first-best stopping time is increasing in ability starting at λ^L, the social surplus from the low type (given by the expression in the first square brackets in (SA.1)) has increasing differences in (T,λ^L), and the low type's expected marginal product given work up to $T+1$, $\overline{\beta}^L_{T+1}\lambda^L$, is increasing in λ^L. Therefore, substituting $\frac{\beta_0\left(1-\lambda^L\right)^T+1-\beta_0}{\left(1-\lambda^L\right)^T\lambda^L} = \frac{\beta_0}{\overline{\beta}^L_{T+1}\lambda^L}$ in (SA.2), we obtain

$$\frac{\partial}{\partial \lambda^L}\left[V(T+1,\lambda^L,\cdot)-V(T,\lambda^L,\cdot)\right] = \delta^{T+1}\left\{\begin{array}{l}\left(1-\mu_0\right)\beta_0\left[\left(1-\lambda^L\right)^T - T\left(1-\lambda^L\right)^{T-1}\left(\lambda^L-c\right)\right] \\ +\mu_0 c\left(1-\lambda^H\right)^T \frac{\beta_0}{\overline{\beta}^L_{T+1}\lambda^L}\left(1+\frac{\lambda^H-\lambda^L}{\overline{\beta}^L_{T+1}\lambda^L}\frac{\partial\left(\overline{\beta}^L_{T+1}\lambda^L\right)}{\partial\lambda^L}\right)\end{array}\right\}>0,$$

which implies that $\bar{t}^L$ is increasing in λ^L.

That $\bar{t}^L$ can increase or decrease in λ^H follows from the fact that (SA.2) yields

$$\begin{aligned}&\frac{\partial}{\partial \lambda^H}\left[V(T+1,\lambda^H,\cdot)-V(T,\lambda^H,\cdot)\right] \\ =&-\delta^{T+1}\mu_0 c\frac{\beta_0\left(1-\lambda^L\right)^T+1-\beta_0}{\left(1-\lambda^L\right)^T\lambda^L}\left[\left(1-\lambda^H\right)^T - T\left(1-\lambda^H\right)^{T-1}\left(\lambda^H-\lambda^L\right)\right],\end{aligned}$$

whose sign can vary with parameters. Specifically, let $(\beta_0,\mu_0,c,\delta,\lambda^L)=(0.95,0.1,0.215,0.8,0.25)$, which results in a first-best stopping time $t^L=5$. Consider three values of λ^H: $\lambda^H_1=0.45$, $\lambda^H_2=0.5$, and $\lambda^H_3=0.55$. The corresponding first-best stopping times are $t^H_1=6$, $t^H_2=5$, and $t^H_3=5$. One can verify that the low type's second-best stopping time, $\bar{t}^L$, increases (from 3 to 4) when λ^H increases from λ^H_1 to λ^H_2 while it decreases (from 4 to 0) when λ^H increases from λ^H_2 to λ^H_3.

Finally, consider the comparative statics of the distortion, $t^L-\bar{t}^L$. By (3), t^L is independent of μ_0 and λ^H, while we have just shown that $\bar{t}^L$ is decreasing in μ_0 and can increase or decrease in λ^H. Therefore, $t^L-\bar{t}^L$ is increasing in μ_0 and can increase or decrease in λ^H depending on parameters. To see that $t^L-\bar{t}^L$ can increase or decrease in β_0 as well, take the set of parameters considered in Figure 2, $(\mu_0,c,\delta,\lambda^L,\lambda^H)=(0.3,0.06,0.5,0.1,0.12)$. The figure shows that given these parameters, $t^L-\bar{t}^L$ decreases (from $12-10=2$ to $15-14=1$) when β_0 increases from 0.85 to 0.89. If instead we take these parameter values but change only μ_0 to $\mu_0=0.7$, we find that the same increase in β_0 leads to an increase in $t^L-\bar{t}^L$ (from $12-1=11$ to $15-1=14$). The comparative static of $t^L-\bar{t}^L$ with respect to c and λ^L can be shown by similar computations.

SA.3. Step 6 of Proof of Theorem 5

We remind the reader that Steps 1–5 of the proof of Theorem 5 are in Appendix C of the paper.

By the previous steps in the proof, we restrict attention to onetime-penalty contracts for the low type such that the low type works in all periods $t \in \{1, \ldots, T^L\}$ and the high type has a most-work optimal stopping strategy. For an arbitrary such contract $\mathbf{C}^L$, let $\hat{t}(\mathbf{C}^L)$ denote the high type's most-work optimal stopping time, i.e. $\hat{t}(\mathbf{C}^L) := \max\{t \in \{1, \ldots, T^L\} : \widehat{a}_s = 1 \text{ for all } s = 1, \ldots, t,\ \widehat{\mathbf{a}} \in \boldsymbol{\alpha}^H(\mathbf{C}^L)\}$. We now show that given T^L, there exists an optimal onetime-penalty contract for the low type $\mathbf{C}^L = (T^L, W_0^L, l_{T^L}^L)$ where $\hat{t}(\mathbf{C}^L)$ is given by

$$T^{HL}(T^L) := \min\left\{t \in \{1, \ldots, T^L\} : \overline{\beta}_{t+1}^H \lambda^H < \overline{\beta}_{T^L}^L \lambda^L \text{ and } (1-\lambda^H)^t \leq (1-\lambda^L)^{T^L}\right\},$$

and $l_{T^L}^L$ is given by

$$\bar{l}_{T^L}^L(T^L) := \min\left\{-\frac{c}{\overline{\beta}_{T^L}^L \lambda^L}, -\frac{c}{\overline{\beta}_{T^{HL}(T^L)}^H \lambda^H}\right\}.$$

When not essential, we suppress the dependence of $\hat{t}(\mathbf{C}^L)$ on $\mathbf{C}^L$. We proceed by proving five claims.

Claim 1: Given any onetime-penalty contract $\mathbf{C}^L = (T^L, W_0^L, l_{T^L}^L)$, $-\overline{\beta}_{\hat{t}+1}^H \lambda^H l_{T^L}^L < c$.

Proof: Suppose to contradiction that $-\overline{\beta}_{\hat{t}+1}^H \lambda^H l_{T^L}^L \geq c$. Then type H is willing to work one more period after having worked for $\hat{t}$ periods, contradicting the definition of $\hat{t}$. ‖

Claim 2: Given an optimal onetime-penalty contract $\mathbf{C}^L = (T^L, W_0^L, l_{T^L}^L)$, $(1-\lambda^H)^{\hat{t}} \leq (1-\lambda^L)^{T^L}$.

Proof: Suppose to contradiction that given an optimal contract $\mathbf{C}^L = (T^L, W_0^L, l_{T^L}^L)$, type H's most-work optimal stopping time $\hat{t}$ is such that $(1-\lambda^H)^{\hat{t}} > (1-\lambda^L)^{T^L}$. Then for any strategy $\widetilde{\mathbf{a}} \in \boldsymbol{\alpha}^H(\mathbf{C}^L)$ where type H works for a total of $\widetilde{t}$ periods, $(1-\lambda^H)^{\widetilde{t}} > (1-\lambda^L)^{T^L}$. Now note that given $\mathbf{C}^L$ and $\widetilde{\mathbf{a}}$, type H's information rent is

$$\beta_0 l_{T^L}^L \left[(1-\lambda^H)^{\widetilde{t}} - (1-\lambda^L)^{T^L}\right] - \beta_0 c \sum_{t=1}^{T^L} \widetilde{a}_t \left[\prod_{s=1}^{t-1}\left(1-\widetilde{a}_s\lambda^H\right) - \left(1-\lambda^L\right)^{t-1}\right]$$
$$+c\sum_{t=1}^{T^L}\left(1-\widetilde{a}_t\right)\left[(1-\beta_0) + \beta_0\left(1-\lambda^L\right)^{t-1}\right].$$

Consider a modification that reduces $l_{T^L}^L$ by $\varepsilon > 0$. By Claim 1, for ε small enough, this modification does not affect incentives, and by $(1-\lambda^H)^{\widetilde{t}} > (1-\lambda^L)^{T^L}$, the modification strictly reduces type H's information rent. But then $\mathbf{C}^L$ cannot be optimal. ‖

Claim 3: In any onetime-penalty contract $\mathbf{C}^L = (T^L, W_0^L, l_{T^L}^L)$, if $\mathbf{1} \in \boldsymbol{\alpha}^L(\mathbf{C}^L)$ then

$$l_{T^L}^L \leq \min\left\{-\frac{c}{\overline{\beta}_{T^L}^L \lambda^L}, -\frac{c}{\overline{\beta}_{\hat{t}(\mathbf{C}^L)}^H \lambda^H}\right\}. \tag{SA.5}$$

Conversely, given any onetime penalty contract $\mathbf{C}^L = (T^L, W_0^L, l_{T^L}^L)$, if $l_{T^L}^L \leq \min\left\{-\frac{c}{\overline{\beta}_{T^L}^L \lambda^L}, -\frac{c}{\overline{\beta}_t^H \lambda^H}\right\}$ for some $t \leq T^L$, then $\hat{t}(\mathbf{C}^L) \geq t$ and $\mathbf{1} \in \boldsymbol{\alpha}^L(\mathbf{C}^L)$.

Proof: For the first part of the claim, assume to contradiction that there is $\mathbf{C}^L = (T^L, W_0^L, l_{T^L}^L)$ such that $\mathbf{1} \in \boldsymbol{\alpha}^L(\mathbf{C}^L)$ but (SA.5) does not hold. Suppose first that $-\frac{c}{\overline{\beta}_{T^L}^L \lambda^L} \leq -\frac{c}{\overline{\beta}_{\hat{t}}^H \lambda^H}$. Then type L is not willing to work for T^L periods; having worked for $T^L - 1$ periods, type L's incentive compatibility constraint for effort in period T^L is $-\overline{\beta}_{T^L}^L \lambda^L l_{T^L}^L \geq c$, which is not satisfied with $l_{T^L}^L > -\frac{c}{\overline{\beta}_{T^L}^L \lambda^L}$. Suppose next that $-\frac{c}{\overline{\beta}_{T^L}^L \lambda^L} > -\frac{c}{\overline{\beta}_{\hat{t}}^H \lambda^H}$. Then type H is not willing to work for $\hat{t}$ periods; having worked for $\hat{t} - 1$ periods, type H is willing to work one more period only if $-\overline{\beta}_{\hat{t}}^L \lambda^H l_{T^L}^L \geq c$, which is not satisfied with $l_{T^L}^L > -\frac{c}{\overline{\beta}_{\hat{t}}^H \lambda^L}$.

For the second part of the claim, assume $l_{T^L}^L \leq \min\left\{-\frac{c}{\overline{\beta}_{T^L}^L \lambda^L}, -\frac{c}{\overline{\beta}_t^H \lambda^H}\right\}$. Consider first type L. The proof is by induction. Consider the last period, T^L. Since no matter the history of effort the current belief is some $\beta_{T^L}^L \geq \overline{\beta}_{T^L}^L$, it is immediate that $-\beta_{T^L}^L \lambda^L l_{T^L}^L \geq c$, and thus it is optimal for type L to work in the last period. Now assume inductively that it is optimal for type L to work in period $t+1 \leq T^L$ no matter the history of effort, and consider period t with belief β_t^L. The inductive hypothesis implies that

$$-\beta_{t+1}^L \lambda^L \left\{ l_{T^L}^L (1-\lambda^L)^{T^L-(t+1)} - c \sum_{s=t+2}^{T^L} \left(1-\lambda^L\right)^{s-(t+2)} \right\} \geq c. \tag{SA.6}$$

Therefore, at period t:

$$-\beta_t^L \lambda^L \left\{ -c + (1-\lambda^L) \left[l_{T^L}^L (1-\lambda^L)^{T^L-(t+1)} - c \sum_{s=t+2}^{T^L} \left(1-\lambda^L\right)^{s-(t+2)} \right] \right\} \geq -\beta_t^L \lambda^L \left[-c + (1-\lambda^L) \left(-\frac{c}{\beta_{t+1}^L \lambda^L} \right) \right] = c,$$

where the inequality uses (SA.6) and the equality uses $\beta_{t+1}^L = \frac{\beta_t^L (1-\lambda^L)}{1-\beta_t^L + \beta_t^L (1-\lambda^L)}$.

Finally, consider type H. By Lemma 3 and the fact that $l_t^L = 0$ for all $t = 1, \ldots, T^L - 1$, type H is indifferent between any two action plans $\mathbf{a}$ and $\mathbf{a}'$ such that $\#\{t : a_t = 0\} = \#\{t : a_t' = 0\}$. Thus, without loss, we restrict attention to stopping strategies, and we only need to show that it is optimal for type H to stop at $s \geq t$. Note that for any $s < t$, given that type H has worked consecutively until and including period s, $-\overline{\beta}_{s+1}^H \lambda^H l_{T^L}^L \geq c$, and thus type H does not want to stop at s. $\|$

Claim 4: There exists an optimal onetime-penalty contract $\mathbf{C}^L = (T^L, W_0^L, l_{T^L}^L)$ satisfying $l_{T^L}^L \geq \min\left\{-\frac{c}{\overline{\beta}_{T^L}^L \lambda^L}, -\frac{c}{\overline{\beta}_{\hat{t}}^H \lambda^H}\right\}$.

Proof: Suppose, to contradiction, the claim is false. Given an optimal onetime-penalty contract for type L, $\mathbf{C}^L = (T^L, W_0^L, l_{T^L}^L)$, and type H's most-work optimal stopping strategy $\widehat{\mathbf{a}}$, type H's information rent is

$$\beta_0 l_{T^L}^L[(1-\lambda^H)^{\hat{t}} - (1-\lambda^L)^{T^L}] - \beta_0 c \sum_{t=1}^{\hat{t}} \left[\left(1-\lambda^H\right)^{t-1} - \left(1-\lambda^L\right)^{t-1}\right]$$
$$+ c \sum_{t=\hat{t}+1}^{T^L} \left[(1-\beta_0) + \beta_0\left(1-\lambda^L\right)^{t-1}\right].$$

Consider a modification that increases $l_{T^L}^L$ by $\varepsilon > 0$. By Claim 4 being false and Claim 3, for ε small enough, working in all periods $t = 1, \ldots, T^L$ remains optimal for type L, and $\widehat{\mathbf{a}}$ remains optimal for type H. But then Claim 2 implies that type H's information rent either goes down or remains unchanged with the modification, and thus there exists an optimal contract $\mathbf{C}^L = (T^L, W_0^L, l_{T^L}^L)$ where the claim is true. ‖

Claim 5: There is an optimal onetime-penalty contract $\mathbf{C}^L = (T^L, W_0^L, l_{T^L}^L)$ with $\hat{t}(\mathbf{C}^L) = T^{HL}(T^L)$.

Proof: Take an arbitrary optimal contract $\mathbf{C}^L = (T^L, W_0^L, l_{T^L}^L)$. By Claims 1 and 3, $\hat{t}(\mathbf{C}^L)$ satisfies $\overline{\beta}_{\hat{t}(\mathbf{C}^L)+1}^H \lambda^H < \overline{\beta}_{T^L}^L \lambda^L$. By Claim 2, $\hat{t}(\mathbf{C}^L)$ satisfies $(1-\lambda^H)^{\hat{t}(\mathbf{C}^L)} \leq (1-\lambda^L)^{T^L}$. Thus, all that remains to be shown is that there exists $\mathbf{C}^L$ where $\hat{t}(\mathbf{C}^L)$ is the smallest period $t \in \{1, \ldots, T^L\}$ that satisfies these two conditions. Suppose to contradiction that this claim is false. Then $\hat{t}(\mathbf{C}^L) - 1$ also satisfies the conditions; that is, $\overline{\beta}_{\hat{t}(\mathbf{C}^L)}^H \lambda^H < \overline{\beta}_{T^L}^L \lambda^L$ and $(1-\lambda^H)^{\hat{t}(\mathbf{C}^L)-1} \leq (1-\lambda^L)^{T^L}$. By Claims 3 and 4, $l_{T^L}^L = \min\left\{-\frac{c}{\overline{\beta}_{T^L}^L \lambda^L}, -\frac{c}{\overline{\beta}_{\hat{t}(\mathbf{C}^L)}^H \lambda^H}\right\}$, and thus since $\overline{\beta}_{\hat{t}(\mathbf{C}^L)}^H \lambda^H < \overline{\beta}_{T^L}^L \lambda^L$, $l_{T^L}^L = -\frac{c}{\overline{\beta}_{\hat{t}(\mathbf{C}^L)}^H \lambda^H} < -\frac{c}{\overline{\beta}_{T^L}^L \lambda^L}$. It follows that type H's incentive constraint in period $\hat{t}(\mathbf{C}^L)$ binds; i.e., type H is indifferent between working and shirking at $\hat{t}(\mathbf{C}^L)$ given that he has worked in all periods $t = 1, \ldots, \hat{t}(\mathbf{C}^L) - 1$ and will shirk in all periods $t = \hat{t}(\mathbf{C}^L) + 1, \ldots, T^L$. Hence, both a stopping strategy that stops at $\hat{t}(\mathbf{C}^L)$ and a stopping strategy that stops at $\hat{t}(\mathbf{C}^L) - 1$ are optimal for type H given $\mathbf{C}^L$, and type H's information rent is the same for either of these two action plans. Type H's information rent can thus be written as

$$\beta_0 l_{T^L}^L[(1-\lambda^H)^{\hat{t}(\mathbf{C}^L)-1} - (1-\lambda^L)^{T^L}] - \beta_0 c \sum_{t=1}^{\hat{t}(\mathbf{C}^L)-1} \left[\left(1-\lambda^H\right)^{t-1} - \left(1-\lambda^L\right)^{t-1}\right]$$
$$+ c \sum_{t=\hat{t}(\mathbf{C}^L)}^{T^L} \left[(1-\beta_0) + \beta_0\left(1-\lambda^L\right)^{t-1}\right].$$

Now consider a modified contract, $\widehat{\mathbf{C}}^L$, obtained from $\mathbf{C}^L$ by increasing $l_{T^L}^L$ by $\varepsilon > 0$. Since $l_{T^L}^L = -\frac{c}{\overline{\beta}_{\hat{t}(\mathbf{C}^L)}^H \lambda^H}$, a stopping strategy that stops at $\hat{t}(\mathbf{C}^L)$ is no longer optimal for type H under $\widehat{\mathbf{C}}^L$. Since $l_{T^L}^L < -\frac{c}{\overline{\beta}_{T^L}^L \lambda^L}$ and $l_{T^L}^L < -\frac{c}{\overline{\beta}_{\hat{t}(\mathbf{C}^L)-1}^H \lambda^H}$, for ε small enough, $\mathbf{1} \in \boldsymbol{\alpha}^L(\widehat{\mathbf{C}}^L)$ and a stopping strategy that stops at $\hat{t}(\mathbf{C}^L) - 1$ remains optimal for type H under $\widehat{\mathbf{C}}^L$. Then $\hat{t}(\widehat{\mathbf{C}}^L) = \hat{t}(\mathbf{C}^L) - 1$, and since $(1-\lambda^H)^{\hat{t}(\mathbf{C}^L)-1} \leq (1-\lambda^L)^{T^L}$, type H's information rent either goes down or remains unchanged with the modification, so $\widehat{\mathbf{C}}^L$ is optimal. If $\hat{t}(\widehat{\mathbf{C}}^L) = T^{HL}(T^L)$, we are done. Otherwise, we can apply the argument to $\hat{t}(\widehat{\mathbf{C}}^L)$ and repeat until we eventually arrive at the desired contract $\mathbf{C}^L$ with $\hat{t}(\mathbf{C}^L) = T^{HL}$. $\|$

SA.4. Details for Subsection 7.1

Here we provide a formal result for the discussion in Subsection 7.1 of the paper.

Theorem 7. *Even if project success is privately observed by the agent, the menus of contracts identified in Theorems 3–6 remain optimal and implement the same outcome as when project success is publicly observable.*

Proof. It suffices to show that in each of the menus, each of the contracts would induce the agent (of either type) to reveal project success immediately when it is obtained. Consider first the menus of Theorem 3 and Theorem 5: for each $\theta \in \{L, H\}$, the contract for type θ, $\mathbf{C}^\theta$, is a penalty contract in which $l_t^\theta \leq 0$ for all t. Hence, no matter which contract the agent takes and no matter his type, it is optimal to reveal a success when obtained. For the implementation in Theorem 4, observe from (8) that type L's bonus contract has the property that $\delta b_{t+1}^L \leq b_t^L$ for all $t \in \{1, \ldots, \bar{t}^L - 1\}$; moreover, this property also holds in type L's bonus contract in Theorem 6 and in type H's bonus contracts in both Theorem 4 and Theorem 6, as these contracts are constant-bonus contracts. Hence, under all these contracts, it is optimal for the agent of either type to disclose success immediately when obtained. *Q.E.D.*

SA.5. Details for Subsection 7.2

Here we provide a formal result for the discussion in Subsection 7.2 of the paper.

Theorem 8. *Assume $t^H > t^L$, $\delta = 1$, and that all transfers must be non-negative. In any optimal menu of contracts, each type $\theta \in \{L, H\}$ is induced to work for some number of periods, $\bar{t}_{\ell\ell}^\theta$, where $\bar{t}_{\ell\ell}^L \leq \bar{t}_{\ell\ell}^H$. Relative to the first-best stopping times, t^H and t^L, the second best has $\bar{t}_{\ell\ell}^H \leq t^H$ and $\bar{t}_{\ell\ell}^L \leq t^L$. The principal can implement the second best using a bonus contract for type H, $\mathbf{C}^H = (\bar{t}_{\ell\ell}^H, W_0^H, \boldsymbol{b}^H)$, and a constant-bonus contract for type L, $\mathbf{C}^L = (\bar{t}_{\ell\ell}^L, W_0^L, b^L)$, such that*

1. *$b^L = \frac{c}{\bar{\beta}^L_{\bar{t}^L_{\ell\ell}} \lambda^L}$;*

2. *Type H gets a rent: $U_0^H(\mathbf{C}^H, \boldsymbol{\alpha}^H(\mathbf{C}^H)) > 0$;*

3. *If $\bar{t}_{\ell\ell}^L > 0$, type L gets a rent: $U_0^L(\mathbf{C}^L, \boldsymbol{\alpha}^L(\mathbf{C}^L)) > 0$;*

4. *$\mathbf{1} \in \boldsymbol{\alpha}^H(\mathbf{C}^H)$; $\mathbf{1} \in \boldsymbol{\alpha}^L(\mathbf{C}^L)$; and $\mathbf{1} = \boldsymbol{\alpha}^H(\mathbf{C}^L)$.*

Proof. The principal's program is the following, called [$\mathrm{P}_{\ell\ell}$]:

$$\max_{(\mathbf{C}^H,\mathbf{C}^L,\mathbf{a}^H,\mathbf{a}^L)} \mu_0 \Pi_0^H\left(\mathbf{C}^H,\mathbf{a}^H\right) + (1-\mu_0)\,\Pi_0^L\left(\mathbf{C}^L,\mathbf{a}^L\right) \tag{$\mathrm{P}_{\ell\ell}$}$$

subject to, for all $\theta, \theta' \in \{L, H\}$,

$$\mathbf{a}^\theta \in \boldsymbol{\alpha}^\theta(\mathbf{C}^\theta), \tag{IC_a^θ}$$

$$U_0^\theta(\mathbf{C}^\theta,\mathbf{a}^\theta) \geq 0, \tag{IR^θ}$$

$$U_0^\theta(\mathbf{C}^\theta,\mathbf{a}^\theta) \geq U_0^\theta(\mathbf{C}^{\theta'},\boldsymbol{\alpha}^\theta(\mathbf{C}^{\theta'})), \tag{$\mathrm{IC}^{\theta\theta'}$}$$

$$W_0^\theta, b_t^\theta, l_t^\theta \geq 0 \text{ for all } t \in \Gamma^\theta. \tag{$\ell\ell^\theta$}$$

Note that the limited liability constraint for type θ, ($\ell\ell^\theta$), implies that this type's participation constraint, (IR^θ), is satisfied. From now on, we thus ignore the constraints (IR^θ).

Step 1: Bonus contracts

We show that it is without loss to focus on bonus contracts. Suppose by contradiction that in the solution to [$\mathrm{P}_{\ell\ell}$], for some $\theta \in \{L, H\}$, $\mathbf{C}^\theta = (\Gamma^\theta, W_0^\theta, \boldsymbol{b}^\theta, \boldsymbol{l}^\theta)$ is not a bonus contract, i.e. $l_t^\theta \neq 0$ for some $t \in \Gamma^\theta$. We can construct an equivalent bonus contract $\widetilde{\mathbf{C}}^\theta = (\Gamma^\theta, \widetilde{W}_0^\theta, \widetilde{\boldsymbol{b}}^\theta)$ as in the proof of Proposition 1:

(a) For any $t \in \Gamma^\theta$, $\widetilde{b}_t^\theta = b_t^\theta - \sum_{s \geq t, s \in \Gamma^\theta} l_s^\theta$,

(b) $\widetilde{W}_0^\theta = W_0^\theta + \sum_{t \in \Gamma^\theta} l_t^\theta$.

Note that by the limited liability constraint, $\mathbf{C}^\theta$ has $W_0^\theta \geq 0$ and $l_t^\theta \geq 0$ for all $t \in \Gamma^\theta$. Hence, $\widetilde{\mathbf{C}}^\theta$ has $\widetilde{W}_0^\theta \geq 0$. Moreover, if $\widetilde{b}_t^\theta < 0$ for some $t \in \Gamma^\theta$, then regardless of his type, the agent shirks in period t under contract $\widetilde{\mathbf{C}}^\theta$. Therefore, we can define another bonus contract, $\widehat{\mathbf{C}}^\theta = (\widehat{\Gamma}^\theta, \widetilde{W}_0^\theta, \widetilde{\boldsymbol{b}}^\theta)$, where $t \in \widehat{\Gamma}^\theta$ if and only if $t \in \Gamma^\theta$ and $\widetilde{b}_t^\theta \geq 0$. Since under contract $\widetilde{\mathbf{C}}^\theta$ the agent of either type receives zero with probability one in all periods t in which $\widetilde{b}_t^\theta < 0$, the incentives for effort for both agent types and the payoffs for the principal and both agent types are unchanged in the new contract $\widehat{\mathbf{C}}^\theta$ in which the agent is locked out in these periods. It follows that the bonus contract $\widehat{\mathbf{C}}^\theta$ is equivalent to contract $\widetilde{\mathbf{C}}^\theta$ and thus to the original contract $\mathbf{C}^\theta$, and it satisfies limited liability.

Step 2: Both types always work

We show that it is without loss to focus on bonus contracts in which each type is prescribed to work in every period under his own contract. Suppose that there is a solution to [$\mathrm{P}_{\ell\ell}$] in which, for some $\theta \in \{L, H\}$, $\mathbf{C}^\theta = \left(\Gamma^\theta, W_0^\theta, \boldsymbol{b}^\theta\right)$ induces $\mathbf{a}^\theta \neq \mathbf{1}$. Consider contract $\widehat{\mathbf{C}}^\theta = \left(\widehat{\Gamma}^\theta, W_0^\theta, \boldsymbol{b}^\theta\right)$ where $t \in \widehat{\Gamma}^\theta$ if and only if $t \in \Gamma^\theta$ and $a_t^\theta = 1$. Notice that in any period t in which type θ shirks under contract $\mathbf{C}^\theta$, he receives zero with probability one; this is the same type θ receives under contract $\widehat{\mathbf{C}}^\theta$ where he is locked out in period t. It follows that the incentives for effort for type θ and both the principal's payoff from type θ and type θ's payoff do not change with the new contract. Moreover, observe that for type $\theta' \neq \theta$, no matter which action he would take at t in any optimal action plan under $\mathbf{C}^\theta$, his payoff from $\widehat{\mathbf{C}}^\theta$ must be weakly lower because the lockout in period t effectively forces him to shirk in period t and receive zero.

Step 3: Connected contracts

It is immediate that given $\delta = 1$, it is without loss to focus on connected bonus contracts: under no discounting, nothing changes when a period $t \notin \Gamma^\theta$ is removed from type θ's bonus contract, $\mathbf{C}^\theta = (\Gamma^\theta, W_0^\theta, \boldsymbol{b}^\theta)$. When a lockout period is removed, the future sequence of transfers and effort is shifted up by one period, but this has no effect on the payoffs of the principal and the agent of either type when there is no discounting.

Step 4: Relaxing the principal's program

By Steps 1-3, we restrict attention to connected bonus contracts that induce each agent type to work in each period under his own contract. We now relax the principal's problem [$\text{P}_{\ell\ell}$] by considering a weak version of (IC^{HL}) in which type H is assumed to exert effort in all periods $t \in \{1, \ldots, T^L\}$ if he takes contract $\mathbf{C}^L$. Ignoring the participation constraints as explained above and denoting the set of connected bonus contracts by $\mathcal{C}^b$, the relaxed program, [$\text{RP}_{\ell\ell}$], is

$$\max_{(\mathbf{C}^H \in \mathcal{C}^b, \mathbf{C}^L \in \mathcal{C}^b)} \mu_0 \Pi_0^H \left(\mathbf{C}^H, \mathbf{1}\right) + (1 - \mu_0)\, \Pi_0^L \left(\mathbf{C}^L, \mathbf{1}\right) \tag{$\text{RP}_{\ell\ell}$}$$

subject to

$$\mathbf{1} \in \boldsymbol{\alpha}^L(\mathbf{C}^L), \tag{IC_a^L}$$
$$\mathbf{1} \in \boldsymbol{\alpha}^H(\mathbf{C}^H), \tag{IC_a^H}$$
$$U_0^L \left(\mathbf{C}^L, \mathbf{1}\right) \geq U_0^L \left(\mathbf{C}^H, \boldsymbol{\alpha}^L(\mathbf{C}^H)\right), \tag{IC^{LH}}$$
$$U_0^H \left(\mathbf{C}^H, \mathbf{1}\right) \geq U_0^H \left(\mathbf{C}^L, \mathbf{1}\right), \tag{Weak-IC^{HL}}$$
$$W_0^L, b_t^L \geq 0 \text{ for all } t \in \{1, \ldots, T^L\}, \tag{$\ell\ell^L$}$$
$$W_0^H, b_t^H \geq 0 \text{ for all } t \in \{1, \ldots, T^H\}. \tag{$\ell\ell^H$}$$

We will solve this relaxed program and later verify that the solution is feasible in (and hence is a solution to) [$\text{P}_{\ell\ell}$].

Step 5: An optimal contract for the low type

Take any arbitrary connected bonus contract $\mathbf{C} = (T, W_0, \boldsymbol{b})$. It follows from Step 3 of the proof of Theorem 3 and the proof of Proposition 1 that type θ's incentive constraint for effort binds in each period $t \in \{1, \dots, T\}$ under contract $\mathbf{C}$ if and only if $\boldsymbol{b} = \overline{\boldsymbol{b}}^{\theta}(T)$, where $\overline{\boldsymbol{b}}^{\theta}(T)$ is defined as follows:

$$\overline{b}_t^{\theta}(T) = \overline{b}^{\theta}(T) := \frac{c}{\overline{\beta}_T^{\theta} \lambda^{\theta}} \text{ for all } t \in \{1, \dots, T\}. \tag{SA.7}$$

We can show that in solving program [RP$_{\ell\ell}$], it is without loss to restrict attention to constant-bonus contracts for type L with bonus as defined in (SA.7). The proof follows from Step 4 in the proof of Theorem 3. Take any arbitrary connected bonus contract $\mathbf{C}^L = (T^L, W_0^L, \boldsymbol{b}^L)$ that induces type L to work in each period $t \in \{1, \dots, T^L\}$. We modify this contract into a constant-bonus contract $\widehat{\mathbf{C}}^L = (T^L, \widehat{W}_0^L, \widehat{b}^L)$ where $\widehat{b}^L = \overline{b}^L(T^L)$ and the modified initial transfer $\widehat{W}_0^L$ is such that $U_0^L(\mathbf{C}^L, \mathbf{1}) = U_0^L(\widehat{\mathbf{C}}^L, \mathbf{1})$. We can show that this modification relaxes (Weak-IC^{HL}) while keeping all other constraints in [RP$_{\ell\ell}$] unchanged, and thus it allows to weakly increase the objective in [RP$_{\ell\ell}$]. We omit the details as the arguments are analogous to those in Step 4 in the proof of Theorem 3.

Step 6: Under-experimentation and positive rents for both types

We first show that the solution to [RP$_{\ell\ell}$] does not induce over-experimentation by either type: $T^L \leq t^L$ and $T^H \leq t^H$. It is useful for our arguments to rewrite the principal's payoff by substituting with (1); we obtain that the objective in [RP$_{\ell\ell}$] can be rewritten as

$$\mu_0 \left\{ \beta_0 \sum_{t=1}^{T^H} \left(1 - \lambda^H\right)^{t-1} \lambda^H \left(1 - b_t^H\right) - W_0^H \right\} + (1 - \mu_0) \left\{ \beta_0 \sum_{t=1}^{T^L} \left(1 - \lambda^L\right)^{t-1} \lambda^L \left(1 - b_t^L\right) - W_0^L \right\}. \tag{SA.8}$$

Suppose per contra that a solution to [RP$_{\ell\ell}$] has a menu of connected bonus contracts $(\mathbf{C}^L, \mathbf{C}^H)$ such that $T^\theta > t^\theta$ for some $\theta \in \{L, H\}$. Without loss by Step 2, $\mathbf{C}^\theta = (T^\theta, W_0^\theta, \boldsymbol{b}^\theta)$ induces type θ to work in each period $t \in \{1, \dots, T^\theta\}$. Note that by the arguments in Step 5, type θ's incentive constraint for effort binds in each period of contract $\mathbf{C}^\theta$ if and only if $b_t^\theta = \frac{c}{\overline{\beta}_{T^\theta}^{\theta} \lambda^\theta}$ for all $t \in \{1, \dots, T^\theta\}$; hence, contract $\mathbf{C}^\theta$ must have $b_t^\theta \geq \frac{c}{\overline{\beta}_{T^\theta}^{\theta} \lambda^\theta}$ for all $t \in \{1, \dots, T^\theta\}$ and $T^\theta > t^\theta$ implies $b_t^\theta > 1$ for all $t \in \{1, \dots, T^\theta\}$. Using (SA.8), this implies that the principal's payoff from type θ is strictly negative if $T^\theta > t^\theta$. But then we can show that there exists a menu of connected bonus contracts that satisfies all the constraints in [RP$_{\ell\ell}$] and yields the principal a strictly larger payoff than the original menu $(\mathbf{C}^L, \mathbf{C}^H)$. This is immediate if the original menu induces both $T^L > t^L$ and $T^H > t^H$, as the principal gets a strictly negative payoff from each type in this case. Suppose instead that the original menu is $(\mathbf{C}^\theta, \mathbf{C}^{\theta'})$ with $T^\theta \leq t^\theta$ for type $\theta \in \{L, H\}$ and $T^{\theta'} > t^{\theta'}$ for $\theta' \neq \theta$. Then consider a menu $(\widehat{\mathbf{C}}^\theta, \widehat{\mathbf{C}}^{\theta'})$ where $\widehat{\mathbf{C}}^\theta = \widehat{\mathbf{C}}^{\theta'} = (T^\theta, 0, \overline{b}^{\theta}(T^\theta))$. This menu trivially satisfies all the constraints in the principal's program. Moreover, compared to the original menu, this menu yields the principal a weakly larger payoff from type θ because it induces this type to work for the same periods as $\mathbf{C}^\theta$ with a (weakly) lower initial transfer and (weakly) lower bonuses in each period $t \in \{1, \ldots, T^\theta\}$, and it yields the principal a strictly larger payoff from type θ' because the payoff from this type under the new menu is non-negative given that the bonus is $\bar{b}^\theta(T^\theta) \leq 1$ in each period $t \in \{1, \ldots, T^\theta\}$.

Next, we show that the solution to [RP$_{\ell\ell}$] yields a positive rent to type H (i.e. $U_0^H(\mathbf{C}^H, \mathbf{1}) > 0$) and it also yields a positive rent to type L (i.e. $U_0^L(\mathbf{C}^L, \mathbf{1}) > 0$) if type L is not excluded. By the limited liability constraints ($\ell\ell^L$) and ($\ell\ell^H$), $U_0^L(\mathbf{C}^L, \mathbf{1}) \geq 0$ and $U_0^H(\mathbf{C}^H, \mathbf{1}) \geq 0$. Moreover, given limited liability, $U_0^\theta(\mathbf{C}^\theta, \mathbf{1}) = 0$ for a type $\theta \in \{L, H\}$ implies $T^\theta = 0$. Hence, if type θ is not excluded, this type receives a strictly positive rent. All that is left to be shown is that the solution to [RP$_{\ell\ell}$] cannot exclude type H, and thus it always yields $U_0^H(\mathbf{C}^H, \mathbf{1}) > 0$. First, suppose that $U_0^L(\mathbf{C}^L, \mathbf{1}) > 0$ and $U_0^H(\mathbf{C}^H, \mathbf{1}) = 0$. Then since $\bar{\beta}_t^H \lambda^H > \bar{\beta}_t^L \lambda^L$ for all $t \leq t^L$ (by the assumption that $t^H > t^L$) and $T^L \leq t^L$, it follows that $U_0^H(\mathbf{C}^L, \mathbf{1}) > U_0^L(\mathbf{C}^L, \mathbf{1}) > 0 = U_0^H(\mathbf{C}^H, \mathbf{1})$, and thus (Weak-IC^{HL}) is violated. Next, suppose that $U_0^\theta(\mathbf{C}^\theta, \mathbf{1}) = 0$ for both types $\theta \in \{L, H\}$. Then $T^\theta = 0$ for both types $\theta \in \{L, H\}$ and the principal's payoff is zero. However, the principal can then strictly improve upon this menu by using a menu of constant-bonus contracts $\widehat{\mathbf{C}}^L = \widehat{\mathbf{C}}^H = (1, 0, \bar{b}^H(1))$, where note that $\bar{b}^H(1) < 1$.

Step 7: The high type experiments more than the low type

We show that the solution to [RP$_{\ell\ell}$] must have $T^L \leq T^H$. Suppose per contra that the solution is a menu of connected bonus contracts $\{\mathbf{C}^L, \mathbf{C}^H\}$ such that $T^L > T^H$. Without loss by Step 5, let $\mathbf{C}^L = (T^L, W_0^L, \bar{b}^L(T^L))$. Note that by (Weak-IC^{HL}), $U_0^H(\mathbf{C}^H, \mathbf{1}) \geq U_0^H(\mathbf{C}^L, \mathbf{1})$. Moreover, by Step 6, $T^L \leq t^L$, which in turn implies $T^L < t^H$. But then it is immediate that a menu $(\widetilde{\mathbf{C}}^L, \widetilde{\mathbf{C}}^H)$ where $\widetilde{\mathbf{C}}^L = \widetilde{\mathbf{C}}^H = (T^L, 0, \bar{b}^L(T^L))$ yields the same amount of experimentation by type L, strictly more efficient experimentation by type H, and payoffs $U_0^L(\widetilde{\mathbf{C}}^L, \mathbf{1}) \leq U_0^L(\mathbf{C}^L, \mathbf{1})$ and $U_0^H(\widetilde{\mathbf{C}}^H, \mathbf{1}) \leq U_0^H(\mathbf{C}^H, \mathbf{1})$, while satisfying all the constraints in [RP$_{\ell\ell}$]. It follows that $(\widetilde{\mathbf{C}}^L, \widetilde{\mathbf{C}}^H)$ yields a strictly larger payoff to the principal than the original menu $(\mathbf{C}^L, \mathbf{C}^H)$, which therefore cannot be optimal.

Step 8: Back to the original problem

We now show that the solution to the relaxed program [RP$_{\ell\ell}$] is feasible and thus a solution to the original program [P$_{\ell\ell}$]. Recall that (given Steps 1-3) the only relaxation in program [RP$_{\ell\ell}$] relative to [P$_{\ell\ell}$] is that [RP$_{\ell\ell}$] imposes (Weak-IC^{HL}) instead of (IC^{HL}). Thus, all we need to show is that given a constant-bonus contract $\mathbf{C}^L = (T^L, W_0^L, \bar{b}^L(T^L))$ with length $T^L \leq t^L$, it would be optimal for type H to work in each period $1, \ldots, T^L$. The claim follows from Step 6 in the proof of Theorem 3 and the proof of Proposition 1. *Q.E.D.*

SA.6. Details for Subsection 7.3

Here we provide details for the discussion in Subsection 7.3 of the paper.

Assume $\beta_0 = 1$ and for simplicity that there is some finite time, $\overline{T}$, at which the game ends. Since $\overline{\beta}_t^{\theta} = 1$ for all $\theta \in \{L, H\}$ and $t \in \{1, \ldots, \overline{T}\}$, the high type always has a higher expected marginal product than the low type, i.e. $\overline{\beta}_t^H \lambda^H = \lambda_H > \overline{\beta}_t^L \lambda^L = \lambda_L$ for all t. Consequently, the methodology used in proving Theorem 3 can be applied, with the conclusions that if the optimal length of experimentation for the low type is some T (constrained to be no larger than $\overline{T}$), the optimal penalty contract for the low type is given by the analog of (6) with $\overline{\beta}_t^L = 1$ for all t:

$$l_t^L = \begin{cases} -\left(1-\delta\right)\frac{c}{\lambda^L} & \text{if } t < T, \\ -\frac{c}{\lambda^L} & \text{if } t = T, \end{cases}$$

and the portion of the principal's payoff that depends on T is given by the analog of (SA.1) with the simplification of $\beta_0 = 1$:

$$\begin{aligned} \widehat{V}(T) &= (1-\mu_0)\sum_{t=1}^{T}\delta^t\left(1-\lambda^L\right)^{t-1}\left(\lambda^L-c\right) \\ &\quad -\mu_0\left\{\begin{array}{l} -\frac{c}{\lambda^L}\sum\limits_{t=1}^{T-1}\delta^t\left(1-\delta\right)\left[\left(1-\lambda^H\right)^t-\left(1-\lambda^L\right)^t\right]-\frac{c}{\lambda^L}\delta^T\left[\left(1-\lambda^H\right)^T-\left(1-\lambda^L\right)^T\right] \\ -\sum\limits_{t=1}^{T}\delta^t c\left[\left(1-\lambda^H\right)^{t-1}-\left(1-\lambda^L\right)^{t-1}\right] \end{array}\right\}. \end{aligned}$$

Hence, for any $T \in \{0, \ldots, \overline{T}-1\}$ we have the following analog of (SA.2):

$$\widehat{V}(T+1)-\widehat{V}(T) = \delta^{T+1}\left[\left(1-\mu_0\right)\left(1-\lambda^L\right)^T\left(\lambda^L-c\right)-\mu_0\frac{c}{\lambda^L}\left(1-\lambda^H\right)^T\left(\lambda^H-\lambda^L\right)\right].$$

Clearly, $\widehat{V}(T+1)-\widehat{V}(T) > (<) 0$ if and only if

$$\left(\frac{1-\lambda^L}{1-\lambda^H}\right)^T > (<) \frac{\mu_0 c\left(\lambda^H-\lambda^L\right)}{\left(1-\mu_0\right)\left(\lambda^L-c\right)\lambda^L}.$$

Since the left-hand side above is strictly increasing in T, it follows that $\widehat{V}(T)$ is maximized by $\bar{t}^L \in \{0, \overline{T}\}$. Hence, whenever it is optimal to have the low type experiment for any positive amount of time, it is optimal to have the low type experiment until $\overline{T}$, no matter the value of $\overline{T}$. Note that whenever exclusion is optimal (i.e. $\bar{t}^L = 0$) when $\beta_0 = 1$, it would also be optimal for all $\beta_0 \leq 1$; this follows from the comparative static of $\bar{t}^L$ with respect to β_0 in Proposition 2.