Proof of Proposition 4, Step 1

Here we provide the derivation for

\[ A_{11}(0, p) > A_1(0, p) = A_2(0, p) = 0, \]  

(23)

where

\[ A(\mu, p) = 2\mu(1-\rho)\rho \int_{s \notin S(B(\mu), p)} (s - \bar{s}(B(\mu), p)) \gamma(s; \mu) ds + \rho^2 \int_{s \notin S(B(\mu), p)} (s - \bar{s}(B(\mu), p))^2 \gamma(s; \mu) ds. \]  

(22)

Claim 1. \( A_1(0, p) = 0. \)

Proof. Differentiate (22) to get

\[ A_1(\mu, p) = 2\mu(1-\rho)\frac{\partial}{\partial \mu} \int_{s \notin S(B(\mu), p)} (s - \bar{s}(B(\mu), p)) \gamma(s; \mu) ds \]

\[ + 2\rho (1-\rho) \int_{s \notin S(B(\mu), p)} (s - \bar{s}(B(\mu), p)) \gamma(s; \mu) ds \]

\[ + \rho^2 \frac{\partial}{\partial \mu} \int_{s \notin S(B(\mu), p)} (s - \bar{s}(B(\mu), p))^2 \gamma(s; \mu) ds. \]  

(A-1)

Let us evaluate this expression at \( \mu = 0. \) The first term is obviously 0. The second term is also 0 because \( B(0) = 0 \) and \( S(0, p) \) is a measure zero set (there is full disclosure when \( \mu = 0 \)). To see that the
third term is also 0, observe that

\[
\frac{\partial}{\partial \mu} \int_{s \in S(B(\mu), p)} (s - \bar{s}(B(\mu), p))^2 \gamma(s; \mu) \, ds
\]

\[
= \frac{\partial}{\partial \mu} \left\{ \int_{s \in \mathbb{R}} (s - \bar{s}(B(\mu), p))^2 \gamma(s; \mu) \, ds - \int_{s \in S(B(\mu), p)} (s - \bar{s}(B(\mu), p))^2 \gamma(s; \mu) \, ds \right\}
\]

\[
= \frac{\partial}{\partial \mu} \left\{ \mu^2 + \sigma_1^2 + \sigma_0^2 - 2\mu \bar{s}(B(\mu), p) + (\bar{s}(B(\mu), p))^2 - \int_{\bar{s}(B(\mu), p) - 2 \frac{B(\mu)}{p}} (s - \bar{s}(B(\mu), p))^2 \gamma(s; \mu) \, ds \right\}
\]

\[
= 2\mu - 2\bar{s}(B(\mu), p) + 2 \bar{s}(B(\mu), p) \bar{s}_1(B(\mu), p) B'(\mu) + \int_{\bar{s}(B(\mu), p) - 2 \frac{B(\mu)}{p}} \gamma(s; \mu) \, ds
\]

\[
+ \left( \bar{s}_1(B(\mu), p) B'(\mu) - 2 \frac{B'(\mu)}{\rho} \right) \left( -2 \frac{B(\mu)}{\rho} \right)^2 \gamma \left( \bar{s}(B(\mu), p) - 2 \frac{B(\mu)}{\rho}; \mu \right),
\]

which is 0 when \( \mu = 0 \), because \( B(0) = 0 \) and \( \bar{s}(0, p) = 0 \). □

**Claim 2.** \( A_{11}(0, p) > 0 \).

**Proof.** Differentiating (A-1) yields

\[
A_{11}(\mu, p) = 2\rho (1 - \rho) \frac{\partial}{\partial \mu} \int_{s \notin S(B(\mu), p)} (s - \bar{s}(B(\mu), p))^2 \gamma(s; \mu) \, ds
\]

\[
+ 2\mu \rho (1 - \rho) \frac{\partial^2}{\partial \mu^2} \int_{s \notin S(B(\mu), p)} (s - \bar{s}(B(\mu), p))^2 \gamma(s; \mu) \, ds
\]

\[
+ 2\rho (1 - \rho) \frac{\partial}{\partial \mu} \int_{s \notin S(B(\mu), p)} (s - \bar{s}(B(\mu), p))^2 \gamma(s; \mu) \, ds
\]

\[
+ \rho^2 \frac{\partial^2}{\partial \mu^2} \int_{s \notin S(B(\mu), p)} (s - \bar{s}(B(\mu), p))^2 \gamma(s; \mu) \, ds.
\] (A-2)

Let us evaluate this expression at \( \mu = 0 \). The second term is obviously 0. For the first and third terms, we have

\[
\frac{\partial}{\partial \mu} \int_{s \notin S(B(\mu), p)} (s - \bar{s}(B(\mu), p))^2 \gamma(s; \mu) \, ds
\]

\[
= \frac{\partial}{\partial \mu} \left\{ \int_{s \in \mathbb{R}} (s - \bar{s}(B(\mu), p))^2 \gamma(s; \mu) \, ds - \int_{s \in S(B(\mu), p)} (s - \bar{s}(B(\mu), p))^2 \gamma(s; \mu) \, ds \right\}
\]

\[
= \frac{\partial}{\partial \mu} \left\{ \mu - \bar{s}(B(\mu), p) - \int_{\bar{s}(B(\mu), p) - 2 \frac{B(\mu)}{p}} (s - \bar{s}(B(\mu), p))^2 \gamma(s; \mu) \, ds \right\}
\]

\[
= 1 - \bar{s}_1(B(\mu), p) B'(\mu) + \int_{\bar{s}(B(\mu), p) - 2 \frac{B(\mu)}{p}} \bar{s}_1(B(\mu), p) B'(\mu) \gamma(s; \mu) \, ds
\]

\[
+ \left( \bar{s}_1(B(\mu), p) B'(\mu) - 2 \frac{B'(\mu)}{\rho} \right) \left( -2 \frac{B(\mu)}{\rho} \right) \gamma \left( \bar{s}(B(\mu), p) - 2 \frac{B(\mu)}{\rho}; \mu \right),
\]
which is equal to 1 at \( \mu = 0 \) because \( \bar{s}_1(0, p) = 0 \) (Step 3 in proof of Proposition 1).

Finally, for the fourth term in (A-2), we have

\[
\frac{\partial^2}{\partial \mu^2} \int_{s \in S(B(\mu), p)} (s - \bar{s}(B(\mu), p))^2 \gamma(s; \mu) \, ds
\]

\[
= \frac{\partial}{\partial \mu} \left\{ 2\mu - 2\bar{s}(B(\mu), p) + 2\bar{s}(B(\mu), p) \bar{s}_1(B(\mu), p) B'(\mu) \right\}
\]

\[
= 2 \left( 1 - \bar{s}_1(B(\mu), p) B'(\mu) + B'(\mu) \left( \bar{s}(B(\mu), p) \bar{s}_{11}(B(\mu), p) B'(\mu) + (\bar{s}_1(B(\mu), p))^2 B'(\mu) \right) \right)
\]

\[-2 \int_{s \in S(\mu), p} \bar{s}_1(B(\mu), p) B'(\mu) \gamma(s; \mu) \, ds
\]

\[-2 \left( \bar{s}_1(B(\mu), p) B'(\mu) - 2 \frac{B'(\mu)}{\rho} \right) \left( -2 \frac{B(\mu)}{\rho} \right) \gamma \left( \bar{s}(B(\mu), p) - 2 \frac{B(\mu)}{\rho}; \mu \right) \, ds
\]

\[+ \frac{4}{\rho^2} 2B(\mu) B'(\mu) \left( \bar{s}_1(B(\mu), p) B'(\mu) - 2 \frac{B'(\mu)}{\rho} \right) \gamma \left( \bar{s}(B(\mu), p) - 2 \frac{B(\mu)}{\rho}; \mu \right)
\]

\[+ \left( -2 \frac{B(\mu)}{\rho} \right)^2 \frac{\partial}{\partial \mu} \left\{ \left( \bar{s}_1(B(\mu), p) B'(\mu) - 2 \frac{B'(\mu)}{\rho} \right) \gamma \left( \bar{s}(B(\mu), p) - 2 \frac{B(\mu)}{\rho}; \mu \right) \right\},
\]

which is equal to 2 at \( \mu = 0 \) because \( B(0) = \bar{s}(0, p) = \bar{s}_1(0, p) = 0 \).

Since \( \rho \in (0, 1) \), it follows that (A-2) is strictly positive, as desired.

**Claim 3.** \( A_2(0, p) = 0 \).

**Proof.** Differentiate (22) to get

\[
A_1(\mu, p) = 2\mu \rho (1 - \rho) \frac{\partial}{\partial \rho} \int_{s \in S(\mu, p)} (s - \bar{s}(B(\mu), p)) \gamma(s; \mu) \, ds
\]

\[+ \rho^2 \frac{\partial}{\partial \rho} \int_{s \in S(\mu, p)} (s - \bar{s}(B(\mu), p))^2 \gamma(s; \mu) \, ds.
\]

(A-3)

Let us evaluate this at \( \mu = 0 \). To see that the first term is 0, observe that

\[
\frac{\partial}{\partial \rho} \int_{s \in S(\mu, p)} (s - \bar{s}(B(\mu), p)) \gamma(s; \mu) \, ds
\]

\[
= \frac{\partial}{\partial \rho} \left\{ \int_{s \in \mathbb{R}} (s - \bar{s}(B(\mu), p)) \gamma(s; \mu) \, ds - \int_{s \in S(\mu, p)} (s - \bar{s}(B(\mu), p)) \gamma(s; \mu) \, ds \right\}
\]

\[
= \frac{\partial}{\partial \rho} \left\{ \mu - \bar{s}(B(\mu), p) - \int_{s \in S(\mu, p), \rho} (s - \bar{s}(B(\mu), p)) \gamma(s; \mu) \, ds \right\}
\]

\[
= -\bar{s}_2(B(\mu), p) - \int_{s \in S(\mu, p), \rho} (-\bar{s}_2(B(\mu), p)) \gamma(s; \mu) \, ds
\]

\[+ (\bar{s}_2(B(\mu), p)) \left( -2 \frac{B(\mu)}{\rho} \right) \gamma \left( \bar{s}(B(\mu), p) - 2 \frac{B(\mu)}{\rho}; \mu \right),
\]
which is equal to \(-\bar{s}_2 (0, p)\) when \(\mu = 0\) because \(B (0) = \bar{s} (0, p) = 0\); and in turn, \(\bar{s}_2 (0, p) = 0\) since \(\bar{s} (0, p) = 0\) for all \(p\).

To see that the second term in (A-3) is 0 at \(\mu = 0\), observe that

\[
\frac{\partial}{\partial p} \int_{s \in S(B(\mu), p)} (s - \bar{s} (B (\mu), p))^2 \gamma (s; \mu) \, ds
\]

\[
= \frac{\partial}{\partial p} \left\{ \mu^2 + \sigma_1^2 + \sigma_0^2 - 2 \bar{s} (B (\mu), p) + (\bar{s} (B (\mu), p))^2 - \int_{s \in S(B(\mu), p)} (s - \bar{s} (B (\mu), p))^2 \gamma (s; \mu) \, ds \right\}
\]

\[
= 2 \mu \bar{s}_2 (B (\mu), p) + 2 (\bar{s} (B (\mu), p)) \bar{s}_2 (B (\mu), p)
\]

\[
- \int_{s \in S(B(\mu), p) - B(\mu)} 2 (s - \bar{s} (B (\mu), p)) (-\bar{s}_2 (B (\mu), p)) \gamma (s; \mu) \, ds
\]

\[
+ (\bar{s}_2 (B (\mu), p)) \left( -2 \frac{B (\mu)}{\rho} \right)^2 \gamma (\bar{s} (B (\mu), p) - 2 \frac{B (\mu)}{\rho}; \mu),
\]

which is 0 at \(\mu = 0\), because \(B (0) = \bar{s} (B (0), p) = 0\).

**Proof of Proposition 4, Step 2**

Here we verify that

\[
v_1 (0, p) = v_2 (0, p) = v_{11} (0, p) = w_1 (0, p) = w_2 (0, p) = w_{11} (0, p) = 0,
\]

(25)

where we had defined

\[
w(\mu, p) := -\bar{s}^2 - \int_{s \in S(B(\mu), p)} (a_q (B (\mu), p) - s \rho)^2 \gamma (s; 0) \, ds,
\]

and

\[
v(\mu, p) := -\bar{s}^2 - \int_{s \in S(B(\mu), p)} (a_q (B (\mu), p) - s \rho)^2 \gamma (s; 0) \, ds.
\]

First, since \(a_q (B (\mu), p) = \rho \bar{s} (B (\mu), p)\),

\[
v_1 (\mu, p) = - \int_{-\infty}^{\infty} 2 (\rho \bar{s} (B (\mu), p) - s \rho) \rho \bar{s}_1 (B (\mu), p) B' (\mu) \gamma (s; 0) \, ds,
\]

(A-4)

\[
v_2 (\mu, p) = - \int_{-\infty}^{\infty} 2 (\rho \bar{s} (B (\mu), p) - s \rho) \rho \bar{s}_2 (B (\mu), p) \gamma (s; 0) \, ds.
\]

Since \(B (0) = \bar{s}_1 (0, p) = \bar{s}_2 (0, p) = 0\), it follows that \(v_1 (0, p) = v_2 (0, p) = 0\).
Second, noting also that $s(B(\mu), p) = \bar{s}(B(\mu), p) - 2\frac{B(\mu)}{\rho}$,  

\[ w_1(\mu, p) = -\int_{\bar{s}(B(\mu), p) - 2\frac{B(\mu)}{\rho}}^{\tau(B(\mu), p)} 2(\rho \bar{s}(B(\mu), p) - \rho s) \rho \bar{s}_1(B(\mu), p) B'(\mu) \gamma(s; 0) ds 
+ \left( \bar{s}_1(B(\mu), p) B'(\mu) - 2\frac{B'(\mu)}{\rho} \right) \left( -2\frac{B(\mu)}{\rho} \right)^2 \gamma \left( \bar{s}(B(\mu), p) - 2\frac{B(\mu)}{\rho}; 0 \right), \quad (A-5) \]

\[ w_2(\mu, p) = -\int_{\bar{s}(B(\mu), p) - 2\frac{B(\mu)}{\rho}}^{\tau(B(\mu), p)} 2(\rho \bar{s}(B(\mu), p) - \rho s) \rho \bar{s}_2(B(\mu), p) \gamma(s; 0) ds 
+ \bar{s}_2(B(\mu), p) \left( -2\frac{B(\mu)}{\rho} \right)^2 \gamma \left( \bar{s}(B(\mu), p) - 2\frac{B(\mu)}{\rho}; 0 \right). \]

Since $B(0) = \bar{s}(0, p) = \bar{s}_2(0, p) = 0$, it follows that $w_1(0, p) = w_2(0, p) = 0$.

Third, differentiating $(A-4)$ yields 

\[ v_{11}(\mu, p) = -2\rho^2 (1 - \rho) \frac{\partial}{\partial \mu} \int_{-\infty}^{\infty} (\bar{s}(B(\mu), p) - s) \bar{s}_1(B(\mu), p) \gamma(s; 0) ds 
\times -\bar{s}_1(B(\mu), p) \int_{-\infty}^{\infty} (\bar{s}(B(\mu), p) - s) \gamma(s; 0) ds - (1 - \rho) \int_{-\infty}^{\infty} (\bar{s}_1(B(\mu), p))^2 \gamma(s; 0) ds, \quad (A-6) \]

where the second line has ignored the factor $2\rho^2 (1 - \rho)$. Let us evaluate $(A-6)$ at $\mu = 0$: since $B(0) = \bar{s}(0, p) = 0$, the first integral term is $\int_{-\infty}^{\infty} s\gamma(s; 0) ds = 0$, while the second integral term is also 0 since $\bar{s}_1(0, p) = 0$. Therefore, $v_{11}(0, p) = 0$.

Fourth, differentiating $(A-5)$ yields 

\[ w_{11}(\mu, p) = -2\rho^2 (1 - \rho) \bar{s}_{11}(B(\mu), p) \frac{\partial}{\partial \mu} \int_{\bar{s}(B(\mu), p) - 2\frac{B(\mu)}{\rho}}^{\tau(B(\mu), p)} (\bar{s}(B(\mu), p) - s) \gamma(s; 0) ds 
-2\rho^2 (1 - \rho)^2 \int_{\bar{s}(B(\mu), p) - 2\frac{B(\mu)}{\rho}}^{\tau(B(\mu), p)} (\bar{s}_1(B(\mu), p))^2 \gamma(s; 0) ds 
+ \frac{\partial}{\partial \mu} \left[ \left( \bar{s}_1(B(\mu), p) B'(\mu) - 2\frac{B'(\mu)}{\rho} \right) \left( -2\frac{B(\mu)}{\rho} \right)^2 \gamma \left( \bar{s}(B(\mu), p) - 2\frac{B(\mu)}{\rho}; 0 \right) \right] \]

\[ = -2\rho^2 (1 - \rho) \bar{s}_{11}(B(\mu), p) \frac{\partial}{\partial \mu} \int_{\bar{s}_1(B(\mu), p) - 2\frac{B(\mu)}{\rho}}^{\tau(B(\mu), p)} (\bar{s}(B(\mu), p) - s) \gamma(s; 0) ds 
-2\rho^2 (1 - \rho)^2 \int_{\bar{s}_1(B(\mu), p) - 2\frac{B(\mu)}{\rho}}^{\tau(B(\mu), p)} (\bar{s}_1(B(\mu), p))^2 \gamma(s; 0) ds 
+ \frac{4}{\rho^2} 2B(\mu) B'(\mu) \left( \bar{s}_1(B(\mu), p) B'(\mu) - 2\frac{B'(\mu)}{\rho} \right) \gamma \left( \bar{s}(B(\mu), p) - 2\frac{B(\mu)}{\rho}; 0 \right) 
+ \left( \frac{-2B(\mu)}{\rho} \right)^2 \frac{\partial}{\partial \mu} \left[ \left( \bar{s}_1(B(\mu), p) B'(\mu) - 2\frac{B'(\mu)}{\rho} \right) \gamma \left( \bar{s}(B(\mu), p) - 2\frac{B(\mu)}{\rho}; 0 \right) \right]. \quad (A-7) \]

Let us evaluate $(A-7)$ at $\mu = 0$. Since $B(0) = \bar{s}(0, p) = 0$, the first two terms have integrals over measure zero sets, hence are 0. Moreover, $B(0) = 0$ implies that the second two terms are also 0. Therefore, $w_{11}(0, p) = 0$. 

5
Proof of Proposition 8

Here we show that \( g'(0) = 0 \) while \( g''(0) = p''(0)(\sigma_0^2 - \bar{\sigma}^2)(\lambda - (f'(0))^2) \), where

\[
g(\mu) := \lambda ( -p(\mu)\tilde{\sigma}^2 - (1 - p(\mu))\sigma_0^2 ) - c(p(\mu) + (p(\mu) - p(f(\mu)))\sigma_0^2 - \bar{\sigma}^2 + \mu^2 - (B(\mu))^2).
\]

Taking the first derivative yields

\[
g'(\mu) = \lambda (\sigma_0^2 - \bar{\sigma}^2) p'(\mu) - c'(p(\mu)) p'(\mu) + (\sigma_0^2 - \bar{\sigma}^2) [p'(\mu) - p'(f(\mu)) f'(\mu)]
+ (p'(\mu) - p'(f(\mu)) f'(\mu)) \left( \mu^2 - B(\mu)^2 \right)
+ (p(\mu) - p(f(\mu))) 2 \left( \mu - B(\mu) (1 - \rho) \right),
\]

and because \( p'(0) = 0 = f(0) \), we have \( g'(0) = 0 \).

Taking the second derivative yields

\[
g''(\mu) = \lambda (\sigma_0^2 - \bar{\sigma}^2) p''(\mu) - p''(\mu) c'(p(\mu)) - c''(p(\mu)) (p'(\mu))^2
+ (\sigma_0^2 - \bar{\sigma}^2) \left[ p''(\mu) - p''(f(\mu)) (f'(\mu))^2 - p'(f(\mu)) f''(\mu) \right]
+ \left[ p''(\mu) - p''(f(\mu)) (f'(\mu))^2 - p'(f(\mu)) f''(\mu) \right] \left( \mu^2 - (B(\mu))^2 \right)
+ (p'(\mu) - p'(f(\mu)) f'(\mu)) 2 \left( \mu - B(\mu) (1 - \rho) \right) + [p'(\mu) - p'(f(\mu)) f'(\mu)] 2 \left( \mu - B(\mu) (1 - \rho) \right)
+ (p(\mu) - p(f(\mu))) 2 \left( 1 - \rho \right).$$

Evaluating at \( \mu = 0 \), and using the facts that \( p'(0) = 0 = f(0) \), we get

\[
g''(0) = \lambda (\sigma_0^2 - \bar{\sigma}^2) p''(0) - p''(0) c'(p(0)) + (\sigma_0^2 - \bar{\sigma}^2) \left[ p''(0) - p''(0) (f'(0))^2 \right],
\]

and now using the fact that \( c'(p(0)) = \sigma_0^2 - \bar{\sigma}^2 \) (by the first-order condition for optimality of \( p(0) \)), we further simplify to

\[
g''(0) = \lambda (\sigma_0^2 - \bar{\sigma}^2) p''(0) - p''(0) (\sigma_0^2 - \bar{\sigma}^2) + (\sigma_0^2 - \bar{\sigma}^2) p''(0) - (\sigma_0^2 - \bar{\sigma}^2) p''(0) (f'(0))^2
= p''(0) (\sigma_0^2 - \bar{\sigma}^2) \left( \lambda - (f'(0))^2 \right).
\]