Sequential Veto Bargaining with Incomplete Information

S. Nageeb Ali  Navin Kartik  Andreas Kleiner
Veto bargaining important in politics & orgs

- Legislatures send bills to Executives
- Executives need legislatures to confirm appointments
- Search committees put forward candidates to their higher-ups
- Boards of Directors require sign-off from shareholders

“If Congress returns the bill having appropriately addressed these concerns, I will sign it. For now, I must veto the bill.”
Veto Bargaining

Veto bargaining: (bilateral) bargaining with single-peaked prefs and one-sided offers

- Proposer and Vetoer
- 1-dimensional policy

Romer and Rosenthal (1978)

- TIOLI offer with complete information
- Proposer targets Vetoer precisely
  → no vetoes, but Vetoer’s ideal point affects outcome, even if she doesn’t obtain any surplus
This Paper

Analysis omits two (related) features:

- Proposer doesn’t know Vetoer’s ideal point

  → **Cannot target precisely**

- Sequential proposals

  → **Proposer can learn from past rejections**

  → **But Vetoer may now strategically reject**

◊ How much does Proposer benefit from sequential proposals?

◊ **Does lack of commitment (significantly) hurt Proposer?**

  *Coasian Conjecture*: Proposer cannot avoid moderating proposals after rejection, so much so that he is at the mercy of Vetoer’s private info

Results

- **Commitment payoff is achievable**

- Such eqa exploit *leapfrogging*
  
  → owes to single-peaked prefs
  
  → unlike usual monopolist

- Other eqa can coexist
  
  → with Coasian dynamics
Existing Work

Sequential veto bargaining
- Romer & Rosenthal 1979; Cameron 2000; Rosenthal & Zame 2019; Chen 2021
- Cameron & Elmes 1995; Evdokimov 2022

Coase Conjecture in seller-buyer settings
- FLT 1985; GSW 1985; AD 1989

Non-Coasian logic in seller-buyer settings
- Board & Pycia 2014; Tirole 2016
- Wang 1998; Hahn 2006; Inderst 2008
Model
Model

At each $t = 0, 1, \ldots$, Proposer makes a proposal $a_t \in \mathbb{R}$ that Vetoer can accept or reject.

Game ends when Vetoer accepts.

If agreement is reached in period $T$, payoffs are

$$\delta^T u(a_T) \quad \text{and} \quad \delta^T u_V(a_T, v)$$

- Until agreement, flow utility from status quo, $a = 0$; normalize this utility to 0.
- After agreement, flow utility from $a_T$.
- So utility measured as gain over status quo.

**Single-peaked preferences**

- Proposer’s ideal point known to be 1.
- Vetoer’s ideal point is $v$, her type, which is private info.

Study PBE.

Nb: can interpret Vetoer as a voting group, so long as Proposer only observes outcome, not vote profile.
Example
Two-Type Example

Proposer \( u(a, v) = 1 - |1 - a| \) (constants normalize \( u(0) = u_V(0, v) = 0 \))

Vetoer \( u_V(a, v) = v - |v - a| \)

Vetoer type \( v \in \{l, h\} \), with \( 0 < l < 1/2 < h < 2l < 1 \)

Under complete information, \( a(h) = 1 \) and \( a(l) = 2l \)

But this violates IC for \( h \)

Proposer’s optimal delegation set (deterministic static mechanism) is either

- Pooling menu \( \{2l\} \)
- Separating menu \( \{a^*, 1\} \), with \( h \) indiff between \( a^* \) and 1

\( \checkmark \) more interesting case
The Sequential Rationality Problem

In our dynamic game without commitment, when players are patient, can Proposer obtain action 1 from type $h$ and $a^*$ from $l$?

Standard “skimming” recipe:

- Propose 1 at $t = 0$, which is accepted by $h$
- If rejected, propose $a^*$ at $t = 1$, which is accepted by $l$

(perhaps modulo some discounting adjustments)

But not an equilibrium!

- After rejection at $t = 0$, Proposer believes Vetoer type is $l$
- Sequential Rationality $\implies$ at $t = 1$ propose $2l > a^*$
- But anticipating $2l$, type $h$ rejects 1 at $t = 0$
The Leapfrogging Solution

Modulo discounting adjustments:

**First** propose $a^*$ (receives $!$ in chess annotation)

- Accepted only by type $l$

**Only then** propose $1$ forever

- Accepted by type $h$

**Key idea:** By first securing agreement with $l$, sequential rationality no longer impels Proposer to moderate should $h$ subsequently reject

**Owes to single-peaked Vetoer prefs**

$\rightarrow$ Futile in monopoly pricing; indeed, all equilibria there have skimming
A Non-Constructive Argument

Result (Two types)
Assume the optimal delegation set has separation. When players are patient, Proposer can achieve (at least) approximately the delegation payoff.

Proof:
Let $a^\delta$ be lowest action s.t. $h$ is indifferent between $a^\delta$ today and 1 tomorrow.

Note that $a^\delta \rightarrow a^*$ as $\delta \rightarrow 1$.

- If Proposer proposes $a^\delta$ in first period, $l$ accepts and $h$ rejects. After rejection of $a^\delta$, Proposer believes $Pr(h) = 1$ and proposes 1 forever.
- If Proposer proposes $a \neq a^\delta$ in first period, play some continuation equilibrium.
- In first period, Proposer chooses an optimal proposal.

So either Proposer uses $(a^\delta, 1)$ on path, or follows another path that is even better.
An Equilibrium Construction

An equilibrium construction is quite involved

<table>
<thead>
<tr>
<th>Natural construction</th>
<th>Potential deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>■ First propose $a^\delta$</td>
<td>■ First offer a high action</td>
</tr>
<tr>
<td>■ If rejected, propose 1 ever after</td>
<td>■ Type $h$ may accept, and Proposer may be better off</td>
</tr>
<tr>
<td>■ Type $l$ accepts $a^\delta$ and type $h$ accepts 1</td>
<td></td>
</tr>
</tbody>
</table>

Resolved by **Proposition 1**, which distinguishes three cases:

(a) **Skimming.** $P(h)$ low: skimming approximates the pooling outcome, which is optimal.

(b) **Leapfrogging.** $P(h)$ moderate: on path offers $(a^\delta, 1)$.

(c) **Delayed leapfrogging.** $P(h)$ high: first offer 1; in second period mix between leapfrogging and skimming. Type $h$ mixes in the first period to justify Proposer’s indifference.
Wrap-up of Example

Example illustrates why leapfrogging works

and how it delivers a high payoff by weakening seq rationality constraint

Limitations of example, beyond specificity

■ are there equilibria that attain even higher or lower Proposer payoffs?

■ why is the optimal delegation payoff the right benchmark?

→ commitment in dynamic game?
General Analysis
Payoffs and Types

Proposer’s \( u(a) \) is (weakly) concave with a unique maximum at 1; and \( u(0) = 0 \)

Vetoer’s \( u_V(a, v) \equiv -(a - v)^2 + v^2 \)
- Normalized so that \( u_V(0, v) = 0 \)
- Single-crossing expectational differences (SCED); Kartik, Lee, Rappoport (2019)
- Interval choice: set of types willing to accept any offer is an interval

Vetoer’s type \( v \sim F \in \mathcal{F} \)
- \( \mathcal{F} \): CDFs with density bounded away from 0 and \( \infty \) on an interval support
- Denote support of \( F \) by \( [v, \bar{v}] \)
- \( \bar{v} \leq 1 \) (for simplicity)
Auxiliary Static Problem

Auxiliary static mechanism design problem:

\[ S \equiv \{ m : [\underline{v}, \overline{v}] \rightarrow \Delta(\mathbb{R}) \text{ s.t. IC and IR} \} \quad (+ \text{ integrable; finite mean and variance lotteries}) \]

\[ U(F) \equiv \max_{m \in S} \int u(m(v))dF(v) \quad \text{Proposer’s optimum} \]

- Stochastic mechanisms are allowed
- This problem studied by Kartik, Kleiner, Van Weelden (2021)

Assumption (Interval delegation is optimal)

An interval delegation set \([c^*, 1]\) solves Proposer’s static problem.

- Simple, deterministic mechanism
- Types above \(c^*\) get ideal point, types in \((c^*/2, c^*)\) get \(c^*\), types below \(c^*/2\) get the SQ 0
- KKVW derive sufficient conditions: e.g., \(f\) logconcave and \(u\) linear-quadratic
An Upper Bound

Why is the static problem relevant to our dynamic game?

Lemma (Upper bound on Proposer’s payoff)

Proposer’s payoff from any strategy, given a Vetoer best response, is at most $U(F)$.

Invoking an auxiliary static problem is familiar from seller-buyer bargaining

Here, absent transfers, important that static problem allow for stochastic mechanisms
An Upper Bound

Why is the static problem relevant to our dynamic game?

Lemma (Upper bound on Proposer’s payoff)

Proposer’s payoff from any strategy, given a Vetoer best response, is at most $U(F)$.

Proof idea:

- Time-stamped allocation $(a, t) \mapsto$ static lottery $(a \text{ w.pr. } \delta^t; \ 0 \text{ w.pr. } 1 - \delta^t)$
- Payoff equivalent for Proposer and all Vetoer types
- Because Vetoer is playing a best response, resulting static mechanism is IC and IR

Lemma holds even if game form allowed cheap talk, menus, etc.

Lemma $\implies$ we can refer to $U(F)$ as commitment payoff (at least upper bound on)
Main Result

Theorem (Commitment payoff is achievable)
Assume an equilibrium exists for all $\delta$ and beliefs in $\mathcal{F}$.
When players are patient, $\exists$ eqm with Proposer payoff approx. his commitment payoff.

- Lack of commitment does not hurt Proposer, given his favorite eqm
- Unless $c^* = 1$ ("no compromise"), sequential proposals strictly better than just TIOLI
- Non-Coasian: if $0 < 2\underline{v} < c^*$, Coasian dynamics suggest compromising down to $2\underline{v}$; not seq rational to stop at $c^*$ when there are pos-surplus types for whom $c^*$ is unacceptable
  → note that $\underline{v} > 0$ is the "gap case"
Main Result

Theorem (Commitment payoff is achievable)

Assume an equilibrium exists for all $\delta$ and beliefs in $\mathcal{F}$. When players are patient, $\exists$ eqm with Proposer payoff approx. his commitment payoff.

Proof ideas:

- $[c^*, 1]$ remains an optimal mech $\forall$ beliefs $F_{[\underline{v},c]}$ with $c \geq c^*$ and for $F_{[c^*/2,c^*]}$ (Lemma 2)
  - Uses SCED and interval delegation structure

- If belief is $F_{[\underline{v},c^*]}$, use option to leapfrog to obtain commitment payoff (Lemma 3)
  - Option to follow path of first offering 0 and then $c^*$ forever
  - If all types below $c^*/2$ accept first offer 0, then $c^*$ is an optimal second offer by Lemma 1 (static mech is upper bound) and Lemma 2, given that it is accepted by all remaining types

- More involved: use induction to extend from $F_{[\underline{v},c^*]}$ to $F_{[\underline{v},\overline{v}]}$, applying Lemmas 1 & 2

Note: we do not construct a commitment-payoff eqm (cf. two types)
Coasian Equilibria

So far: maximum Proposer payoff. But can other eqs coexist, perhaps with a Coasian flavor?

**Full Delegation**: interval delegation set \([2 \max\{0, v\}, 1]\)

- Vetoer gets much discretion; if \(v = 0\), every Vetoer type gets her first best
- Proposer only minimally exploiting his bargaining power
  - Caveat: full delegation can sometimes be an optimal mech
Coasian Equilibria

So far: maximum Proposer payoff. But can other eqa coexist, perhaps with a Coasian flavor?

Proposition (Coasian dynamics)
If $v \leq 0$ or $\bar{v} \leq 1/2$, $\exists$ skimming eqm; at patient limit, outcome is full delegation.

- Resolves eqm existence
- Construction adapts “dynamic programming” arguments from seller-buyer analyses
- But single-peakedness necessitates some differences
- When $v > 0$, have to deter low-offer deviations (leapfrogging is salient!);
  $\bar{v} \leq 1/2$ ensures that any such deviation can be accepted by all types, hence unattractive

$\rightarrow$ Norms can matter in veto bargaining: requires sequentiality and incomplete info
Related Literature

Sequential veto bargaining
- Romer & Rosenthal 1979; Cameron 2000; Rosenthal & Zame 2019; Chen 2021
- Cameron & Elmes 1995; Evdokimov 2022

Coase Conjecture in seller-buyer settings
- FLT 1985; GSW 1985; AD 1989

Non-Coasian logic in seller-buyer settings
- Board & Pycia 2014; Tirole 2016
- Wang 1998; Hahn 2006; Inderst 2008
Conclusion
Conclusion

Bilateral bargaining over policy: *single-peaked preferences*
Proposer is *uncertain* of Vetoer’s ideal point, and can make sequential proposals

**Takeaway #1:** Leapfrogging behavior
- First secure agreement with low types
  - weaken subsequent sequential rationality constraints
  - thereby extract surplus from high types
- Absent when dividing a dollar/monopoly pricing

**Takeaway #2:** Commitment payoff can be achieved
- Fundamentally non-Coasian

**Takeaway #3:** Other equilibria can coexist
- Coasian intuition has some merit: full delegation can arise
- Norms can matter
Thank you!