Muddled Information

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Muddled Information

- Signaling games
  - Agent takes (costly) action to affect beliefs about type
- How much information is revealed?
- Standard models: actions (can) fully reveal type
  - single-crossing condition $\implies$ separating equilibrium
  - doesn't vary with parameters
- But in many applications
  - revealed information seems imperfect
  - concern that signaling actually reduces information: "gaming"

Muddled Information

- Model: agents vary on multiple dimensions
  - info on one dimension muddled with that on other
  - generally won't get separation (on any dimension)
- Study: when and how does signaling degrade or improve info?

Signaling, Information, and Gaming

Example: College admissions

- SAT score is a signal of college aptitude
- Students can do test prep to improve score
- Some students are better at test prep than others
- Score muddles info on college aptitude with facility at test prep

- March 2014: College Board announced plans to redesign SAT, in part to "rein in **the intense coaching and tutoring on how to take the test that often gave affluent students an advantage**." (NYT)

Signaling, Information, and Gaming

Example: Google's organic search rankings

- Sites can use SEO to rank higher
- Some sites more eager to use, skilled at, or have resources for SEO
- Google's ranking muddles info on relevance with skill at SEO

- Concern that Google actively tries to deal with:

  All those people who have sort of been doing... "over optimization" or "overly" doing their SEO, compared to the people who are just making great content and trying to make a fantastic site, we want to sort of make that playing field a little bit more level.

  — Matt Cutts, head of Google's Webspam team (2012)

Signaling with Two-Dimensional Types

- Costs of (one-dim) action depend on two-dim type
  - Natural Action
    - Intrinsic ideal point; action absent signaling concerns
  - Gaming Ability
    - Skill at increasing action above natural level
- Each dimension ordered by single-crossing
  - $\uparrow$ natural action or gaming ability $\implies$ $\downarrow$ MC of higher action
- No global ordering due to cross types

Information on Dimension of Interest

- One dimension of interest: **natural action** or gaming ability
- In eqm, info about this dim muddled with that about other
  - Higher actions correlated with higher types, but imperfectly
  - An intermediate action $\rightarrow$ HL, LH, or MM?
- Main results concern comparative statics
  - Uncover key condition on cost function s.t.
  - $\uparrow$ incentive to take higher actions...
    - e.g., $\uparrow$ stakes or making it easier to manipulate signals
  - will tend to $\uparrow$ gaming and
    - $\downarrow$ info about natural action while
    - $\uparrow$ info about gaming ability
- New informational-externality tradeoff
  - showing signal to more markets $\rightarrow$ higher stakes
    $\rightarrow$ degrade informativeness of signal $\rightarrow$ harms existing markets

The Model

Model: The Signaling Game

- Agent has type $\theta = (\eta, \gamma) \in \mathbb{R} \times \mathbb{R}_{++}$
  - η: Natural Action
  - γ: Gaming Ability

- Signaling action $a \in \mathbb{R}$ at cost $C(a, \theta) \equiv C(a, \eta, \gamma)$

- "Market" observes action a, forms beliefs on agent's type
  - Dimension of interest $\tau \in \{\eta, \gamma\}$
  - Expectation $\hat{\tau} \equiv \mathbb{E}[\tau]$

- Signaling benefit $V(\hat{\tau}; s)$
  - Parameter $s > 0$ varies the "stakes" in signaling

- Net payoff to agent: $V(\hat{\tau}; s) - C(a, \eta, \gamma)$

Signaling Benefit

Total payoff $V(\hat{\tau}; s) - C(a, \eta, \gamma)$

- Assumptions on V:
  0. Continuous
  1. For any s, $V(\hat{\tau}; s)$ strictly $\uparrow$ in $\hat{\tau}$
  2. Increasing differences: for $\hat{\tau}' > \hat{\tau}$, $V(\hat{\tau}'; s) - V(\hat{\tau}; s)$ strictly $\uparrow$ in s
  3. For $\hat{\tau}' > \tau$, $V(\hat{\tau}'; s) - V(\hat{\tau}; s) \begin{array}{ll} \to \infty & \text{as } s \to \infty \\ \to 0 & \text{as } s \to 0 \end{array}$

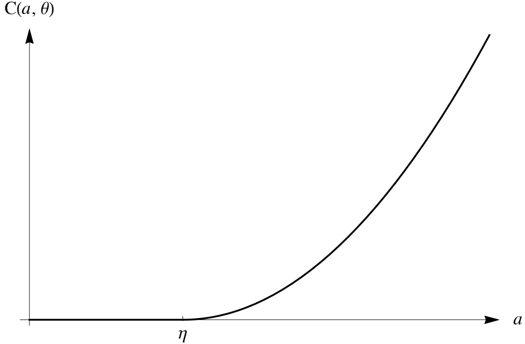
- Leading example: $V(\hat{\tau}; s) = s \cdot v(\hat{\tau})$, where $v(\cdot)$ is strictly $\uparrow$

Signaling Cost

Natural action η, Gaming ability γ, Cost $C(a, \eta, \gamma)$

Assumptions on cost function:

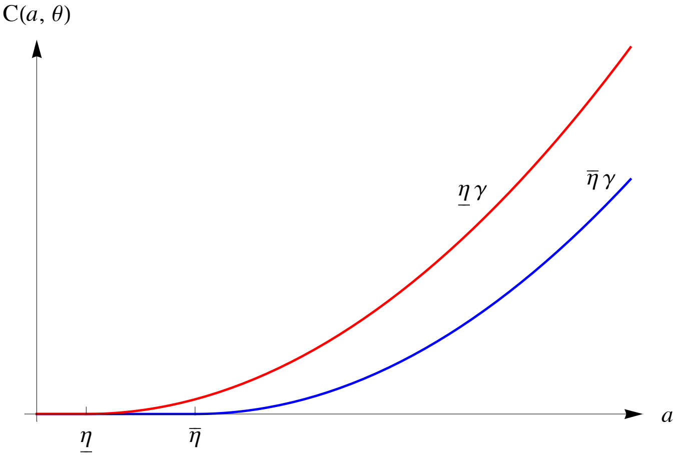
0. C is differentiable; twice-diff. except possibly when $a = \eta$
1. $C(a, \eta, \gamma) = 0$ for $a \leq \eta$
   - Ideal point is η, free downward deviations
2. For $a > \eta$, $C_{aa} > 0$



Signaling Cost

Natural action η, Gaming ability γ, Cost $C(a, \eta, \gamma)$

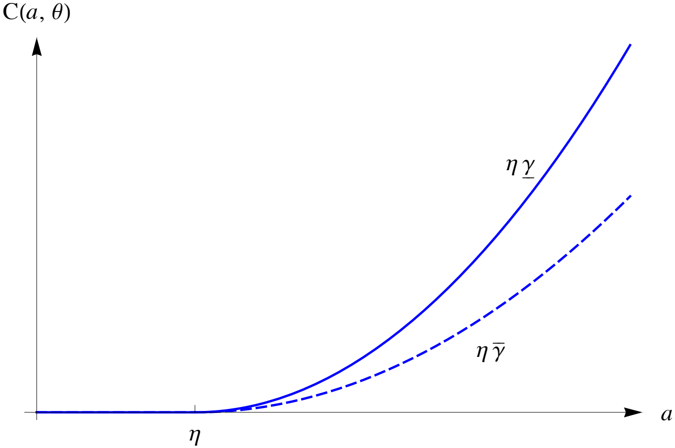
3. For $a > \eta$, $C_{a\eta} < 0$
   - Higher natural action $\implies$ lower MC of $\uparrow a$



Signaling Cost

Natural action η, Gaming ability γ, Cost $C(a, \eta, \gamma)$

4. For $a > \eta$, $C_{a\gamma} < 0$
   - Higher gaming ability $\implies$ lower MC of $\uparrow a$

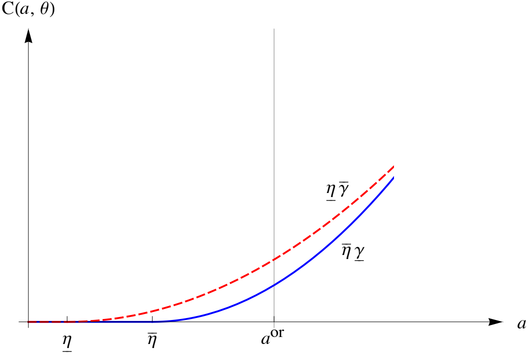


Signaling Cost

Natural action η, Gaming ability γ, Cost $C(a, \eta, \gamma)$

❺ For any pair of **cross types**, $(\overline{\eta}, \underline{\gamma})$ and $(\underline{\eta}, \overline{\gamma})$,

$$\frac{C_a(\cdot, \overline{\eta}, \underline{\gamma})}{C_a(\cdot, \underline{\eta}, \overline{\gamma})} \text{ is str. } \uparrow \text{ on } [\overline{\eta}, \infty) \text{ and } = 1 \text{ at some } a^{\text{or}} > \overline{\eta}.$$



Signaling Cost

Natural action η, Gaming ability γ, Cost $C(a, \eta, \gamma)$

Interpretation of last assumption:

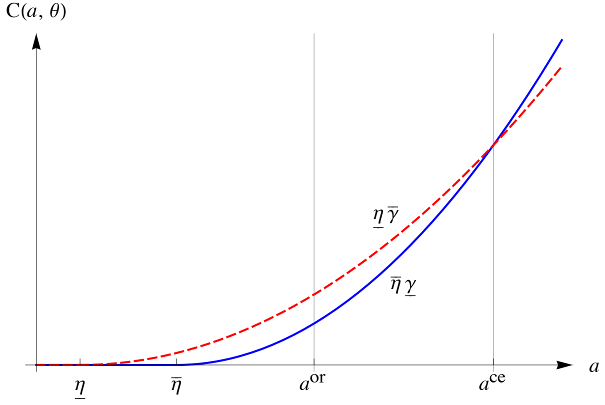
- At higher actions, MC depends relatively more on gaming ability, less on natural action
- For cross types, ordering by MC reverses as we increase actions
- Credit score example
  - Anne has high natural score of 575, low gaming ability
  - Bob has low natural score of 500, high gaming ability
  - At lower credit scores around 600 $\rightarrow$ Anne has lower MC
  - At higher credit scores around 700 $\rightarrow$ Bob has lower MC

Signaling Cost

Natural action η, Gaming ability γ, Cost $C(a, \eta, \gamma)$

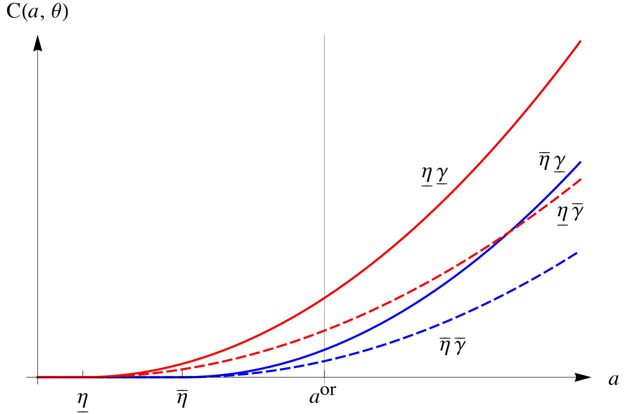
Assms imply:

For any pair of cross types, $\exists$ **cost-equalizing action** $a^{\text{ce}} > a^{\text{or}}$



Signaling Cost

Natural action η, Gaming ability γ



Leading example: $C(a, \eta, \gamma) = \begin{cases} \frac{(a-\eta)^r}{\gamma} & \text{if } a > \eta \\ 0 & \text{o/w} \end{cases}$ with $r > 1$

Nb: $V(\hat{\tau}; s) - c(a, \eta)/\gamma$ equiv. to $\gamma V(\hat{\tau}; s) - c(a, \eta)$

Equilibrium

Total payoff $V(\hat{\tau}; s) - C(a, \eta, \gamma)$

- Fix stakes s and a type-distribution F with compact support Θ
- (Perfect) Bayesian equilibria
  - Market has belief $\hat{\tau}(a)$, given by Bayes rule on path
  - Agent chooses a to $\max V(\hat{\tau}(a); s) - C(a, \eta, \gamma)$
- Pooling equilibrium always exists
  - FDD $\implies$ can pool at lowest natural action
- Interested in (possibility of) equilibria that are informative about τ
  - Partially pooling, or separating on τ

Equilibrium Information

Want to study how equilibrium information about τ changes with stakes

Perspective: welfare (gross of signaling costs) $\uparrow$ with information about τ

Comparing information across equilibria

- Blackwell partial order
- Any eqm e induces a distr. over posteriors on τ, $\beta_e \in \Delta(\Delta(\Theta_\tau))$
- $e \geq e'$ if β_e is a weak MPS of $\beta_{e'}$
- (Or: can compare distributions of $\hat{\tau}$)

Comparing information across sets of equilibria

- Multiple equilibria at given stakes
- Blackwell-induced weak set order to compare eqm sets E and E'
- $E \geq E'$ if
  1. $\forall e \in E$, $\exists e' \in E'$ s.t. $e \geq e'$: [Automatic b/c of pooling eqm]
  2. $\forall e' \in E$, $\exists e \in E$ s.t. $e \geq e'$

Related Literature

- Signaling with one-dimensional types
  - Type = gaming ability: Spence (1973)
  - Type = natural action: Stein (1989), Bernheim (1994), Kartik (2009)
- Multi-dimensional-type signaling models
  - Austen-Smith and Fryer (2005), Bagwell (2007)
  - **Fischer and Verrechia (2000)**, **Bénabou and Tirole (2006), Gesche (2015)**
- "Countersignaling": 1-dim type without single-crossing
  - Feltovich et al (2002), Araujo et al (2007), Chung and Esö (2013)
- Other reasons for incomplete revelation with 1-dim types
  - Bounded action space (Cho and Sobel 1990)
  - Noisy signaling tech (Matthews and Mirman 1983)
  - Equilibrium selection

Intuition: Two Cross Types

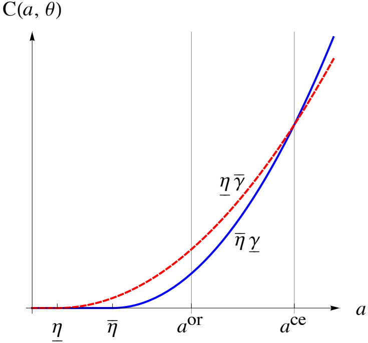
Two Cross Types with $\tau = \eta$

$\theta_1 = (\overline{\eta}, \underline{\gamma})$ and $\theta_2 = (\underline{\eta}, \overline{\gamma})$

Claim (Separation on η only at low stakes)

Let $\tau = \eta$. $\exists\ s^* > 0$ s.t. a separating eqm exists if and only if $s \le s^*$.

- Separating eqm at low stakes
  - Very low stakes: both types choose ideal points $\underline{\eta}$ and $\overline{\eta}$
  - Slightly higher stakes: gamer chooses $\underline{\eta}$, natural chooses $a > \overline{\eta}$

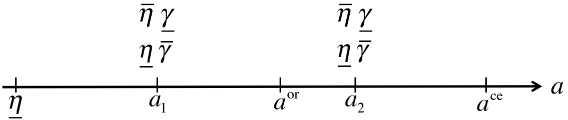


- No separating eqm at high stakes
  - a^{ce} acts like endog. upper bound
  - At high stakes, gamer willing to take a^{ce} to mimic natural

Two Cross Types with $\tau = \eta$

$\theta_1 = (\overline{\eta}, \underline{\gamma})$ and $\theta_2 = (\underline{\eta}, \overline{\gamma})$

- At higher stakes, no eqm is fully informative about η
- For arbitrarily high stakes, $\exists$ partially informative eqa

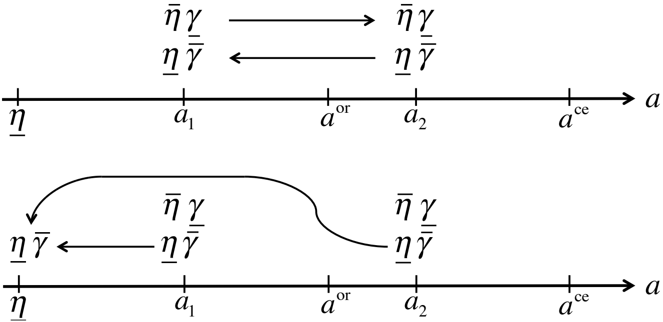


- Both types mix over a_1 and a_2
- Higher beliefs at a_2 because natural more likely to play a_2
- Set mixing probs to get indifference over a_1 and a_2
- As $s \to \infty$, beliefs must converge, information vanishes

Two Cross Types with $\tau = \eta$

$\theta_1 = (\overline{\eta}, \underline{\gamma})$ and $\theta_2 = (\underline{\eta}, \overline{\gamma})$

- At any stakes, some informative equilibrium
- At higher stakes, equilibria are less informative
  - Fix any equilibrium
  - When stakes reduce, can construct a more informative eqm
    - Move some gamer types left, from high to low beliefs
    - Move some natural types right, from low to high beliefs



Two Cross Types with $\tau = \eta$

- Summary when dimension of interest is natural action:
  - Separation at low stakes
  - Information decreases in stakes

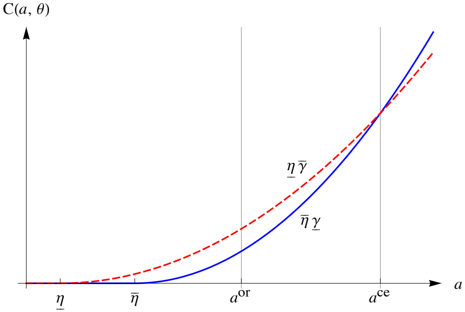
Two Cross Types with $\tau = \gamma$

$\theta_1 = (\overline{\eta}, \underline{\gamma})$ and $\theta_2 = (\underline{\eta}, \overline{\gamma})$

Claim (Separation on γ only at high stakes)

Let $\tau = \gamma$. $\exists$ thresholds $0 < s^{**} < s^*$ s.t. a separating eqm exists if and only if $s \geq s^*$, and only uninformative eqa exist for $s < s^{**}$.

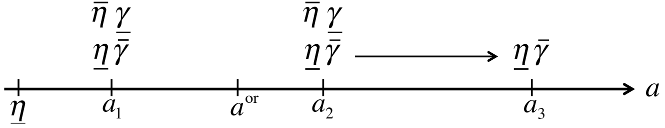
- At high stakes: full information
  - Gamer separates with $a > a^{\text{ce}}$
- At low stakes: no information
  - "High type" (gamer) not willing to pay cost to take an action higher than $\overline{\eta}$
  - "Low type" (natural) willing to take $a \leq \overline{\eta}$ to pool



Two Cross Types with $\tau = \gamma$

$\theta_1 = (\overline{\eta}, \underline{\gamma})$ and $\theta_2 = (\underline{\eta}, \overline{\gamma})$

- Pooling at low stakes, separation at high stakes
- At higher stakes, equilibria are more informative
  - Fix any equilibrium
  - As we increase stakes, can perturb eqm to be more informative
    - Move some gamer types right to a new action



Two Cross Types with $\tau = \gamma$

- Summary when dimension of interest is gaming ability:
  - Separation at high stakes, pooling at low stakes
  - Information increases in stakes

Moving beyond cross types

- Two cross types illustrates intuition, but setting is artificial
  - Type space should embed cross types and ordered types

1. Minimal such setting: 2×2 model
   - $\Theta = \{\underline{\eta}, \overline{\eta}\} \times \{\underline{\gamma}, \overline{\gamma}\}$
   - Exact comparative statics
2. General type spaces
   - Limiting results as stakes get small or large
3. Linear-quadratic-elliptical specification
   - Workhorse functional forms, distributions; explicit (linear) eqm
   - Exact comparative statics in stakes, variances, corr.

2×2 Model

2×2 Model – Main Result

Proposition (Effect of stakes on information)

Consider the 2×2 model and some stakes $\underline{s} < \overline{s}$.

1. If the dimension of interest is the natural action, then the set of equilibria under $\underline{s}$ is more informative than under $\overline{s}$.
2. If the dimension of interest is the gaming ability, then the set of equilibria under $\underline{s}$ is less informative than under $\overline{s}$.

Proof idea: extends the cross-type logic

1. Dimension of interest is $\tau = \eta$:
   - Starting from $\overline{s}$, can continuously perturb any equilibrium to be Blackwell more informative as stakes s decrease
2. Dimension of interest is $\tau = \gamma$:
   - Starting from $\underline{s}$, can continuously perturb any equilibrium to be Blackwell more informative as stakes s increase

General Type Spaces: Limiting Results

Natural Action – Low stakes

Type space Θ with projection Θ_η onto η.

Proposition (Separation on η only at low stakes)

Assume $\tau = \eta$. If $|\Theta_\eta| < \infty$ then:

1. For sufficiently small stakes, there is a separating equilibrium on η.
2. If Θ contains a pair of cross types, then for sufficiently high stakes there is no separating equilibrium on η.

Proof:

1. Construct separating eqm: e.g. agents play $a = \eta$
2. Follows from analysis of cross types:
   - At high stakes, no separating eqm if Θ is just two cross types
   - $\implies$ no separating eqm when embedded in a larger type space

Natural Action – High stakes

Proposition (Vanishing information on η at high stakes)

Assume $\tau = \eta$. If γ and η are independent, and γ has a continuous distribution, then as stakes $s \to \infty$ all eqa become uninformative about η.

- We prove something stronger:

  Given a pair of cross types $\theta_1 = (\underline{\eta}, \overline{\gamma})$ and $\theta_2 = (\overline{\eta}, \underline{\gamma})$ and any $\varepsilon > 0$, for large enough s it holds that $\hat{\eta}_1 \geq \hat{\eta}_2 - \varepsilon$
  - At limit, higher γ types induce weakly higher belief $\hat{\eta}$
  - Limiting distribution of beliefs must be "ironed"
- Why need independence?
  - Positive correlation breaks the result
    - Revealing high $\gamma \implies$ high $\hat{\eta}$
    - At high stakes, high γ types reveal themselves
  - Result does extend to $\mathbb{E}[\eta|\gamma] \downarrow$ in γ
- Why need continuous γ?
  - Extreme types (highest η and γ, or lowest) can always be revealed

Gaming Ability – High stakes

Type space Θ with projection Θ_γ onto γ.

Proposition (Separation on γ only at high stakes)

Assume $\tau = \gamma$. If $|\Theta_\gamma| < \infty$ then:

1. For sufficiently large stakes, there is a separating equilibrium on γ.
2. If Θ contains a pair of cross types, then for sufficiently small stakes there is no separating equilibrium.

Proof:

1. Construct separating equilibrium
2. Follows from analysis of cross types:
    - At low stakes, no separating equilibrium if Θ is just two cross types
    - $\implies$ no separating eqm when embedded in a larger type space

Gaming Ability – Low stakes

Proposition (Vanishing information on γ at low stakes)

Assume $\tau = \gamma$. If γ and η are independent, and η has a continuous distribution, then as stakes $s \to 0$ all eqa become uninformative.

- We prove something stronger:

  Given a pair of cross types $\theta_1 = (\underline{\eta}, \overline{\gamma})$ and $\theta_2 = (\overline{\eta}, \underline{\gamma})$, for small enough s it holds that $\hat{\gamma}_1 \leq \hat{\gamma}_2$

- Independence, continuity assumptions as before
  - extends to $\mathbb{E}[\gamma|\eta] \downarrow$ in η

LQE Specification

- Payoffs: for some $s > 0$, $s\hat{\tau}(a) - \frac{(\max\{a - \eta, 0\})^2}{2\gamma}$
- 2-dim (abs. cont.) Elliptical distribution $\mathcal{E}(\mu, \Sigma, g)$
  - constant density on each concentric ellipse about mean $\mu = (\mu_1, \mu_2)$
  - normal distribution is a special case

▸ Details

- Let $\theta \equiv (\eta, \gamma) \sim \mathcal{E}(\mu_\theta, \Sigma_\theta, g_\theta)$, where
  - $\mu_\theta = (\mu_\eta, \mu_\gamma)$
  - $\Sigma_\theta = \begin{pmatrix} \sigma_\eta^2 & \rho\sigma_\eta\sigma_\gamma \\ \rho\sigma_\eta\sigma_\gamma & \sigma_\gamma^2 \end{pmatrix}$ with $\sigma_\eta > 0$, $\sigma_\gamma > 0$, $\rho \in (-1, 1)$
  - $\gamma > 0$ and compact support $\implies$ $g_\theta(\cdot) : \mathbb{R}_+ \to [0, 1]$ and $\mu_\gamma > \sigma_\gamma$ (ruling out normals)

LQE Specification

Given $\theta \equiv (\eta, \gamma) \sim \mathcal{E}(\mu_\theta, \Sigma_\theta, g_\theta)$

- Marginal of $\tau \in \{\eta, \gamma\}$ depends only on μ_τ and σ_τ^2
- Cov matrix is $\alpha\Sigma$ for some $\alpha > 0$ that depends only on g_θ
- ρ is the correlation coefficient. Nb: $R^2_{\eta\gamma} = \rho^2$
- Crucial property of ellipticals:
  - linear $a(\eta, \gamma) \implies \mathbb{E}[\tau|a]$ is a linear fn of a

    (Quadratic costs and linear benefit is what closes the equilibrium)
- Focus on linear equilibria. In such eqa:
  - the vector $(\tau, a) \sim \mathcal{E}(\cdot, \cdot, g_\theta)$
  - distr. of posteriors β_τ determined entirely by $R^2_{\tau a}$
  - Fixing distr. of τ, higher $R^2_{\tau a} \implies$ Blackwell more info. about $\hat{\tau}$
- Nb: there is always a constant equilibrium (by FDD)

LQN Example: $\tau = \eta$

Proposition

In the LQE specification, assume $\tau = \eta$ and $\rho \geq 0$.

1. There is a unique increasing linear equilibrium.
2. In that equilibrium:
   - $\frac{\mathrm{d}}{\mathrm{d}s} R^2_{\eta a} < 0$, with $R^2_{\eta a}$ going from 1 to ρ^2;
   - $\frac{\mathrm{d}}{\mathrm{d}\mu_\gamma} R^2_{\eta a} = 0$;
   - $\frac{\mathrm{d}}{\mathrm{d}\sigma_\gamma} R^2_{\eta a} < 0$;
   - $\frac{\mathrm{d}}{\mathrm{d}\rho} R^2_{\eta a} > 0$.

LQN Example: $\tau = \gamma$

Proposition

In the LQE specification, assume $\tau = \gamma$ and $\rho \geq 0$.

1. When $\rho = 0$, there is an increasing linear equilibrium if and only if $s > \sigma_\eta^2/\sigma_\gamma^2$; the increasing linear equilibrium is unique when it exists.
2. When $\rho > 0$, there is a unique increasing linear equilibrium.
3. In the increasing linear equilibrium:
   - $\frac{\mathrm{d}}{\mathrm{d}s} R_{\eta a}^2 > 0$, with $R_{\eta a}^2$ going from ρ^2 (if $\rho > 0$) to 1;
   - $\frac{\mathrm{d}}{\mathrm{d}\mu_\eta} R_{\eta a}^2 = 0$;
   - $\frac{\mathrm{d}}{\mathrm{d}\sigma_\eta} R_{\eta a}^2 < 0$;
   - $\frac{\mathrm{d}}{\mathrm{d}\rho} R_{\eta a}^2 > 0$.

Application 1: Manipulability

Manipulability

Reinterpret results on "stakes" as results on "manipulability"

- Consider functional form $sv(\hat{\tau}) - \frac{c(a,\eta)}{\gamma}$
- Add a new parameter Manipulability:

$$sv(\hat{\tau}) - \frac{1}{M}\frac{c(a,\eta)}{\gamma}$$

- Higher M uniformly reduces signaling costs for all agents,
  - More easily manipulated tech.: search engine using keywords vs. links
  - Release information to agents about algorithm
  - Agents become more familiar with a given technology
- Effect of increasing manipulability?
  - Equivalent to increasing stakes s: $Msv(\hat{\tau}) - \frac{c(a,\eta)}{\gamma}$
  - Reduces info on natural action, Increases info on gaming ability

Manipulability

Increasing manipulability M:
Reduces info on natural action

- Fair Isaac keeps FICO score algorithm secret (to extent it can):
  Suggests it's interested in natural action, not gaming ability

  Fair Isaac, the leader in the credit-scoring world, wanted to keep the information secret. The company said it worried that consumers wouldn't understand the nuances of credit scoring, or they would try to 'game the system' if they knew more. **Fair Isaac feared that its formulas would lose their predictive abilities if consumers started changing their behavior to boost their scores.** *(Weston, 2008)*

- Likewise, Google releases "best practices" but keeps details of its ranking algorithm secret

  "[W]e can't divulge specific signals because **we don't want to give people a way to game our search results and worsen the experience** *for users." (Matt Cutts, head of Webspam team)*

Manipulability

- New change to SAT:

  College Board working with Khan Academy to develop free test prep

  $\rightarrow$ Increases manipulability, reduces information??!

  - Free test prep makes gaming *differentially* less costly
    - Rich kids, high γ: Already pay for test prep, no change
    - Poor kids, low γ: Make studying easier

      $\Longrightarrow$ "Levels the playing field"
  - Reducing heterogeneity on γ can increase info about η
    - Limiting case: bring everyone up to same gaming ability

      $\Longrightarrow$ separating equilibrium on η
    - Global comparative static in LQE specification:

      Information on $\hat{\eta}$ is independent of μ_γ, but decreases in σ_γ^2

- Policy recommendation ☺:

  Keep your old tests secret if no current student has them,
  but disclose to all if some students already have them

Application 2: Informational Externalities

Informational Externalities Across Markets

- Adding a new market observer raises stakes
  - Eg, more consumers using ranking system (Google, US News, Yelp)
- Signal on gaming ability gets better
  - "Positive informational externality" –
    New and old observers more informed
- Signal on natural action gets worse
  - "Negative informational externality" –
    New observers more informed
    Old observers less informed
- Should we hide info from some observers to preserve info for others?

Informational Tradeoffs – Motivating example

Credit history relevant to a number of markets

- Main use of credit reports – loan market
  - Help determine how much credit to offer, at what rates
- But also valuable to other markets
- Insurance: good credit $\implies$ file fewer auto, homeowners claims
- Employers: good credit $\implies$ less employee theft, more trustworthy
- Use of credit reports is regulated
  - Various states limit or ban use by employers & some insurance
  - Political arguments — "fairness"
- We offer new argument for (potentially) restricting access:

  Showing credit scores to insurers may degrade loan market efficiency

Informational Tradeoffs over Heterogeneous Markets

- Social value of information measured by **allocative efficiency** ($\mathbb{E}[V]$)
- Two markets in which agent's type is relevant
  1. "Loan market" – information is socially valuable
     - Convex value $v(\hat{\eta})$: info affects allocations
  2. "Insurance market" – information is not socially valuable
     - Linear value $w(\hat{\eta}) = \alpha\hat{\eta}$: info affects prices, not allocations
- Always show action ("credit history") to first market
- Can hide action from insurance market or reveal it
  - If hidden ($s = 1$): $V(\hat{\eta}; 1) = v(\hat{\eta}) + \alpha\mathbb{E}[\eta]$
  - If revealed ($s = 2$): $V(\hat{\eta}; 2) = v(\hat{\eta}) + \alpha\hat{\eta}$
- Revelation only affects allocative efficiency in loan market

Proposition (Sometimes optimal to limit observability)

In the 2×2 model, allocative efficiency is maximized (over $s = 1, 2$ and over equilibria) by hiding information from the insurance market ($s = 1$).

Informational Tradeoffs over Homogeneous Markets

Now suppose **information is equally valuable in all markets**

Should we still hide info from some observers to preserve it for others?

- Let $V(\hat{\eta}; s) = s\,v(\hat{\eta}) = s\big(w(\hat{\eta}) - w(\mathbb{E}[\eta])\big)$ for convex $w(\cdot)$
- Allocative efficiency measured by $s\mathbb{E}[v(\hat{\eta})]$
- Look at canonical case where $\mathbb{E}[v(\hat{\eta})] \to 0$:

Proposition (Homogeneous markets – don't restrict observability)

Let η and γ be independent, with $\eta \in \{0, 1\}$ and γ distributed continuously. Let $C(a, \eta, \gamma) = \frac{(a-\eta)^r}{\gamma}$ for $r > 1$. As $s \to \infty$, there is a sequence of equilibria with allocative efficiency going to infinity.

Take-away:

"Don't hide the action if information is equally valuable across markets"

Proof is constructive:

- Construct sequence of equilibria with $\mathbb{E}[v(\hat{\eta})] \to 0$ at $s^{-\frac{2}{r+1}}$ rate
- Allocative efficiency $s\mathbb{E}[v(\hat{\eta})] \to \infty$ at $s^{\frac{r-1}{r+1}}$ rate

Conclusion

- Amount of info. revealed by signaling is of substantial importance
  - How accurate are Google search results?
  - How predictive are credit scores about loan repayments?
  - How well do SAT scores predict college success?
- This is an endogenous equilibrium quantity
  - What are the benefits to the agents of higher beliefs?
  - What are the costs to the agents of manipulation?
- We provide some new insights on this issue
  - Model: Agents have a natural action and a gaming ability
  - When agents are pushed towards higher actions

    $\because \Delta$ stakes, manipulability, transparency
    - more info on gaming ability
    - less info on natural action
- Implication – there may be informational externalities
  - Showing information to one market may affect info in other markets

Thank you!

LQE Specification

- 2-dim (abs. cont.) Elliptical distribution $\mathcal{E}(\mu, \Sigma, g)$
  - constant density on each concentric ellipse about mean $\mu = (\mu_1, \mu_2)$
  - $\Sigma = \begin{pmatrix} \sigma_1^2 & \sigma_{12} \\ \sigma_{12} & \sigma_2^2 \end{pmatrix}$ a positive definite matrix
  - $g(\cdot) : \mathbb{R}_+ \to \mathbb{R}_+$ measurable fn ("density generator")
  - PDF is
$$f(x) = k|\Sigma|^{-1/2} g((x-\mu)\Sigma^{-1}(x-\mu)'),$$
    for constant of integration $k > 0$

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    for constant of integration $k > 0$
  - normal distribution obtains when $g(t) = \exp\{(-1/2)t\}$

Return