# Lemonade from Lemons: Information Design with Interdependent Values

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# Introduction

- Asymmetric information can affect market outcomes
  - (in)efficiency & distribution
- Various mechanisms can alter—help or hurt—outcomes
- Our paper: information design
  - fix a canonical interdependent-values trading environment
  - characterize **all** outcomes as participants' info varies

    $\rightarrow$ interested in more than just efficiency
- Interpretations
  - designer with some objective (e.g., regulator)
  - predictions across info structures

# Punchlines

- Information design can achieve a lot
  - with no restrictions, all feasible and “indiv. rational” payoffs
  - restrictions to canonical classes of info **do** matter;
    but not in some salient cases

- Methodological contributions
  - allow information to vary on both sides of market
  - identify role of canonical information classes

# Example

# Example (1)

- Seller can sell one indivisible good

| | Prob$(1/2)$ | Prob$(1/2)$ |
|---|---|---|
| Buyer's valuation $v$ | $1$ | $2$ |
| Seller's cost $c(v)$ | $1/2$ | $2$ |

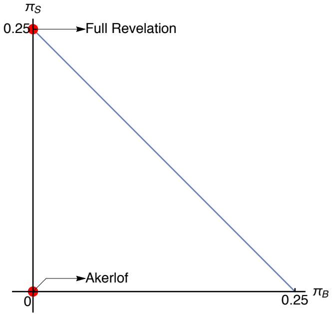
- Seller posts a TIOLI price $p \in \mathbb{R}$

- Payoffs:

| | Seller | Buyer |
|---|---|---|
| No trade | $0$ | $0$ |
| Trade | $p - c(v)$ | $v - p$ |

- Akerlof benchmark: Fully-informed Buyer; Uninformed Seller
  - eqm price $p = 2$ (or $p > 2$); no gains from trade; foregone surplus $1/4$

# Example (2)




- Both informed: eqm price $p = v$; all surplus to Seller

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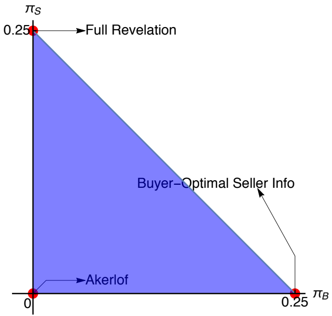
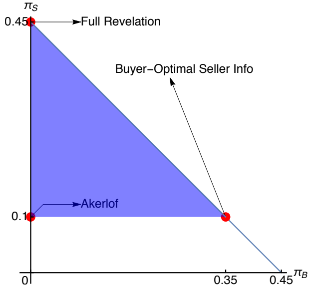



- Both informed: eqm price $p = v$; all surplus to Seller
- $\exists$ Seller info (with informed Buyer) giving all surplus to Buyer?
    - Yes: reveal $c = 2$ sometimes and o-wise induce belief with $\mathbb{E}c = 1$. Upon latter, Seller prices at $1$, efficient trade, no surplus to Seller.
- **All** points in $\triangle$ with some Seller info (and informed Buyer)
- Feasibility + IR $\implies$ nothing else implemented with **any** info design

# Example (3)

| | Prob(1/2) | Prob(1/2) |
|---|---|---|
| Buyer's valuation $v$ | $1$ | $2$ |
| Seller's cost $c(v)$ | $\mathbf{0.3}$ | $\mathbf{1.8}$ |

- Akerlof benchmark: $p = 2$; still inefficient, but some gains from trade



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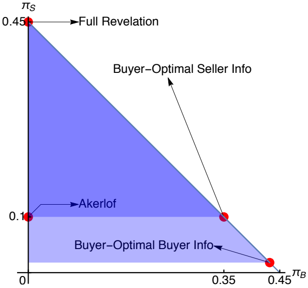
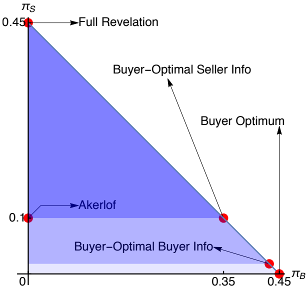



- Implement other payoffs with some Buyer info and uninformed Seller
- In fact, a **superset** of those with fully-informed Buyer

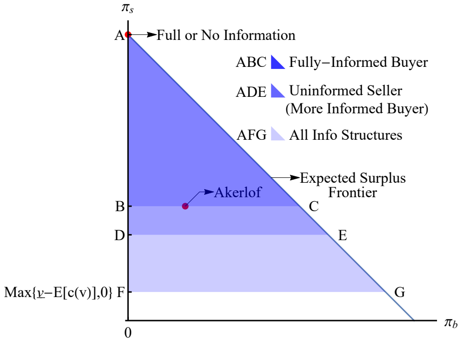
# Example (4)




- Nothing else implementable if Buyer more informed than Seller
- But o-wise can implement still more
  - e.g., Uninformed Buyer; with $\varepsilon$ pr. Seller is informed of $v = 1$ Seller's $p \approx \mathbb{E}c$ indep of signal; Buyer gets approx entire surplus

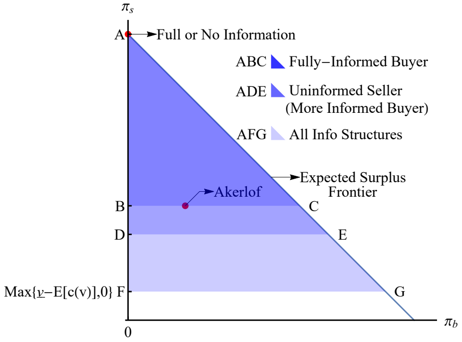
    $\rightarrow$ Seller's info makes off-path belief that $v = 1$ credible
- Using joint info design, can fill in the entire feasible & IR $\triangle$

# General Results




- Uninformed Seller sufficient for more-informed Buyer
  - more generally, if Buyer does not update from price
- All three triangles coincide if and only if either
  - Akerlof info can generate full trade
  - Akerlof info can generate no trade

# Literature




Monopoly pricing

- BBM 2015
- Roesler, Szentes 2017

Info design in games

- Berg, Morris 2016
- Doval, Ely 2020
- Makris, Renou 2021

Others

- Kessler 2001; Levin 2001
- Bar-Isaac, Jewitt, Leaver 2020

# Model

# Model

- Buyer's valuation: $v \in [\underline{v}, \overline{v}]$; prior $\mu$ with support $V$
- Seller's cost: $c(v) \leq v$, continuous with $\mathbb{E}[v - c(v)] > 0$
- Private signals $t_b, t_s \sim P(t_b, t_s | v)$: info structure; design variable
  $\rightarrow$ private signals are wlog
- Seller posts a price $p \in \mathbb{R}$; Buyer decides whether to accept
- Seller, Buyer vNM payoffs: $\begin{cases} (0,0) & \text{if no trade} \\ (p - c(v), v - p) & \text{if trade} \end{cases}$
- weak Perfect Bayesian Equilibrium
  + strengthenings

Nb: not assuming $c(v) \uparrow$

subsumes monopoly pricing, adverse or favorable selection

# Canonical Info Structures and Payoff Sets

$\Gamma \equiv (c(v), \mu)$ is the environment

Canonical information classes

- $\mathbf{T}$: all (joint) info structures
- $\mathbf{T}_{mb}$: Buyer more informed than Seller, i.e., $t_b$ is suff statistic for $v$
- $\mathbf{T}_{us}$: Seller uninformed (singleton signal space)
- $\mathbf{T}_{fb}$: Buyer fully informed of $v$

Implementable payoffs

- $\mathbf{\Pi}(\Gamma)$: payoff vectors across all info structures and all wPBE
- $\mathbf{\Pi}^*(\Gamma)$: subset with price-independent beliefs
  $\rightarrow$ Buyer does not update from price, after conditioning on $t_b$
  $\rightarrow$ implied by NSWYDK if Buyer more informed
- $\mathbf{\Pi}_i^*(\Gamma)$: further subset when information structure is restricted to class $i = mb, us, fb$

$$\mathbf{\Pi}_{us}^*(\Gamma) \cup \mathbf{\Pi}_{fb}^*(\Gamma) \subset \mathbf{\Pi}_{mb}^*(\Gamma) \subset \mathbf{\Pi}^*(\Gamma) \subset \mathbf{\Pi}(\Gamma)$$

# Results

# All Info Structures

Total surplus: $\mathbb{E}[v - c(v)] \equiv S(\Gamma)$

Seller guarantee: $\max\{\underline{v} - \mathbb{E}[c(v)], 0\} \equiv \underline{\pi}_s(\Gamma)$

Buyer guarantee: $0$

**Theorem (All info structures and equilibria.)**

$$\mathbf{\Pi}(\Gamma) = \left\{ (\pi_b, \pi_s) : \begin{array}{l} \pi_b \geq 0 \\ \pi_s \geq \underline{\pi}_s(\Gamma) \\ \pi_b + \pi_s \leq S(\Gamma) \end{array} \right\}.$$

Moreover, $\forall \varepsilon > 0$ $\exists$ a finite information structure and price grid whose set of **sequential equilibrium** payoffs is an $\varepsilon$-net of $\mathbf{\Pi}(\Gamma)$.

Nb: a single information structure implements entire payoff set

# Proof of All-Info Theorem

Assume, for simplicity, $\underline{v} \geq \mathbb{E}[c(v)]$.

- Neither player receives any information
- Seller randomizes between $p_l \in [\underline{v}, \mathbb{E}[v]]$ and $p_h = \mathbb{E}[v]$
  $\rightarrow$ two parameters: $p_l$ and $\sigma(p_l)$

  Buyer accepts $p_l$ but randomizes after $p_h$ to make Seller indifferent

$$\pi_s = p_l - \mathbb{E}[c(v)] \ , \ \pi_b = \sigma(p_l)(\mathbb{E}[v] - p_l)$$

- As $p_l \uparrow$, $\pi_s$ traverses $[\underline{\pi}_s(\Gamma), S(\Gamma)]$
  As $\sigma(p_l) \uparrow$, $\pi_b$ traverses $[0, S(\Gamma) - \pi_s]$
- Off path Buyer belief is $v = \underline{v}$, so Buyer rejects all off-path $p \geq \underline{v}$
- Violates NSWYDK (consider monopoly pricing) ☹
  But can be modified: e.g., if $\Pr(\underline{v}) > 0$, Seller occasionally learns $\underline{v}$
  In fact, get sequential eqm—even "D1"—in discretizations

# More-informed Buyer

$$\underline{\pi}_s^{us}(\Gamma) \equiv \inf\left\{\pi_s : \exists(\pi_b, \pi_s) \in \mathbf{\Pi}^*_{us}(\Gamma)\right\}$$

**Theorem (Equilibria with price-independent beliefs.)**

1. $\mathbf{\Pi}^*(\Gamma) = \mathbf{\Pi}^*_{mb}(\Gamma) = \mathbf{\Pi}^*_{us}(\Gamma)$.
2. $\mathbf{\Pi}^*_{us}(\Gamma) = \left\{(\pi_b, \pi_s) \in \mathbf{\Pi}(\Gamma) : \pi_s \geq \underline{\pi}_s^{us}(\Gamma)\right\}$.
3. $\forall(\pi_b, \pi_s) \in \mathbf{\Pi}^*_{us}(\Gamma)$ with $\pi_s > \underline{\pi}_s^{us}(\Gamma)$, $\exists\tau \in \mathbf{T}_{us}$ s.t. all equilibria have payoffs $(\pi_b, \pi_s)$.

- Given price-indep beliefs, uninformed Seller is sufficient
- Only additional constraint now is $\underline{\pi}_s^{us}(\Gamma) \geq \underline{\pi}_s(\Gamma)$. Inequality is strict if $\underline{v} \leq \mathbb{E}[c(v)]$ and $c(v) < v \ \forall v$.
- Unique implementation

# Price-indep Beliefs Theorem: Proof Sketch

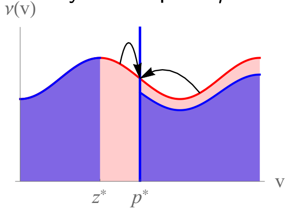
- With price-indep beliefs, $\pi_s \geq \underline{\pi}_s^{us}(\Gamma)$
  - price-indep beliefs $\implies$ info cannot hurt Seller
- Show $\underline{\pi}_s^{us}(\Gamma)$ is implementable with some $\tau^* \in \mathbf{T}_{us}$ (i.e., $\inf = \min$)

**Lemma**

$\forall(\pi_b, \pi_s) \in \mathbf{\Pi}(\Gamma)$ with $\pi_s > \underline{\pi}_s^{us}(\Gamma)$,

$\exists$ garbling of $\tau^*$ s.t. all equilibria have payoffs $(\pi_b, \pi_s)$.

Suppose $\tau^*$ has fully-informed Buyer and prior $\mu$ has density:




Identify $z^*$ and $p^* \in [z^*, \mathbb{E}[v|v > z^*]]$:

- $z^* \leftarrow$ Surplus: $\pi_s + \pi_b = \Pr(v > z^*)\mathbb{E}[v - c(v)|v > z^*]$
- $p^* \leftarrow$ Seller payoff: $\pi_s = \Pr(v > z^*)\mathbb{E}[p^* - c(v)|v > z^*]$

# Fully-Informed Buyer

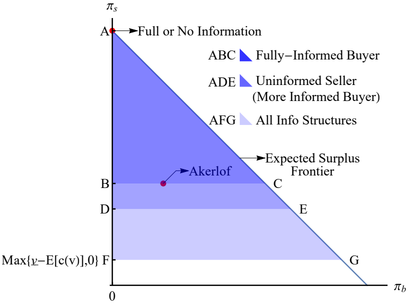
$$\underline{\pi}_s^{fb}(\Gamma) \equiv \sup_p \int_p^{\bar{v}} (p - c(v))\mu(\mathrm{d}v)$$

**Theorem (Fully-informed Buyer, w/ price-indep beliefs.)**

1. $\mathbf{\Pi}^*_{fb}(\Gamma) = \{(\pi_b, \pi_s) \in \mathbf{\Pi}(\Gamma) : \pi_s \geq \underline{\pi}_s^{fb}(\Gamma)\}$.
2. $\forall(\pi_b, \pi_s) \in \mathbf{\Pi}^*_{fb}(\Gamma)$ and $\varepsilon > 0$,
   $\exists \tau \in \mathbf{T}_{fb}$ with all eqm payoffs in $\varepsilon$-ngbhd of $(\pi_b, \pi_s)$.

- Of course, $\underline{\pi}_s^{fb}(\Gamma) \geq \underline{\pi}_s^{us}(\Gamma)$; strictly if $\underline{\pi}_s^{fb}(\Gamma) > \underline{\pi}_s(\Gamma)$
- Proof via "incentive compatible distributons", generalizing Bergemann, Brooks & Morris' (2015) "extreme markets"
- Approx. unique implementation

# In Sum




Extensions/other issues:

- characterizing uninformed-Seller bound $\underline{\pi}_s^{us}$ (✓ linear $v$)
- more general correlation in $c, v$ (✓ if $c \leq v$)
- negative trading surplus (✓ for all info structures; nonlinear frontier)
- other mechanisms
  - if $\underline{v} - \mathbb{E}[c(v)] \leq 0$, cannot implement any more s.t. participation
  - if $\underline{v} - \mathbb{E}[c(v)] > 0$, mech design is useful

**Thank you!**